FOM Instituut voor Plasmafysica Rijnhuizen

Controllability analysis of the magnetic flux distribution in ITER Hybrid scenarios

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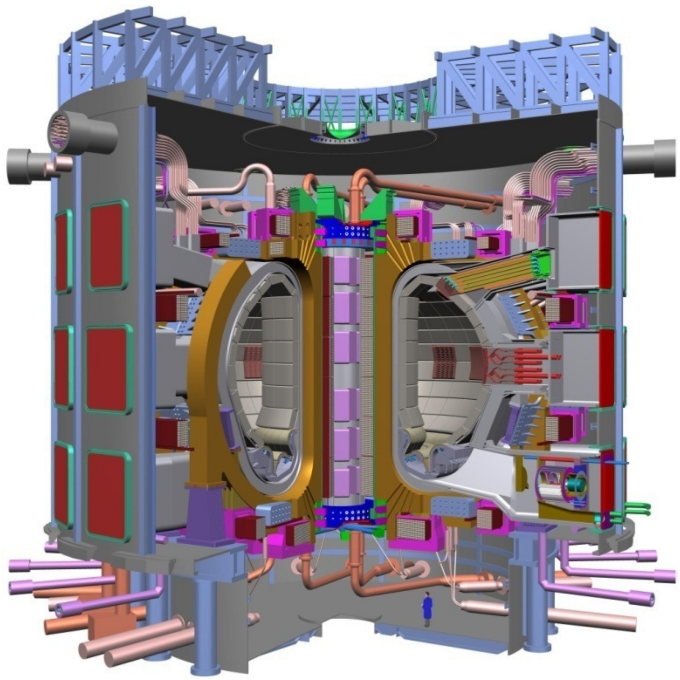
Where innovation starts

Application of CRONOS on ITER scenario

ITER Goals for hybrid' scenario (long discharge time)

- P_{fus} > 350MW
- I_p = 11-13 MA
- $P_{fus} > 5*P_{in}$
- $t_{discharge}$ > 1000 s
- q>1 (for stability)



ITER hybrid scenario primary mission is to maximize neutron fluence per pulse, for reactor-relevant component testing: Long reliable discharge with sufficient neutron flux

CRONOS simulations carried out with the 'GLF23' turbulent transport model

s/q effect on transport tested by simulating various current drive mixes

(J.Citrin, G.M.D.Hogeweij, J.Garcia, J.F. Artaud, F.Imbeaux, Nucl. Fusion 2010)

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Importance of q-profile shaping

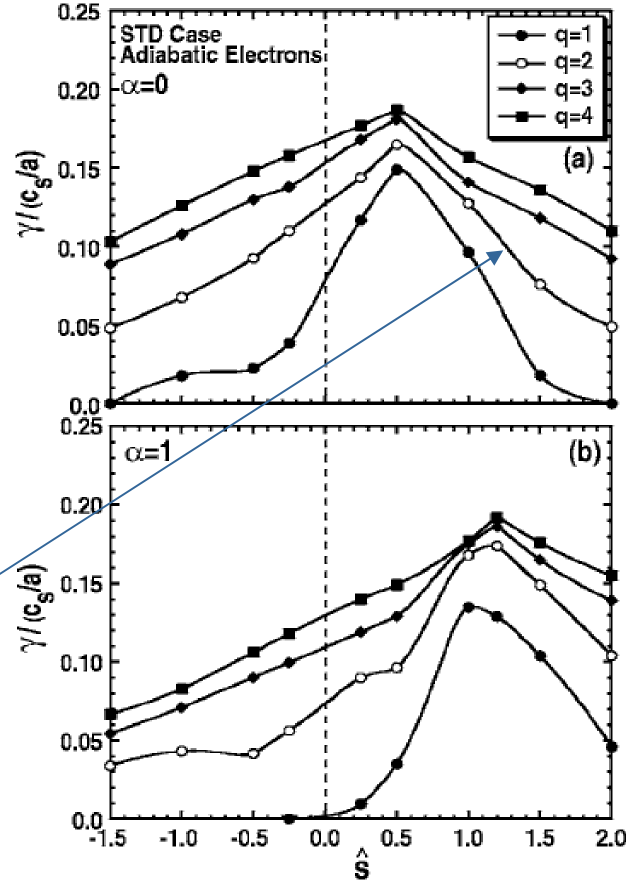
q-profile shape can be **optimized for improved confinement.** Theoretical models– including the GLF23 transport model – predict instability thresholds linearly dependent on s/q (for s>~0.5)

Optimizing the q-profile shape with non-inductive current drive reduces the density necessary for P_{fus}=350MW.

At the same Greenwald fraction, the current can thus also be reduced, decreasing I_{ohm}, and increasing t(q=1).

In the moderate/high magnetic shear regime, we want to maximize s/q.

GYRO ITG linear growth rates



Kinsey, Waltz, Candy (Phys. Plasmas 2006)

Control oriented modeling

The poloidal magnetic flux $\psi(x,t)$ at any point in the poloidal cross section is the total flux through the surface S bounded by the toroidal ring passing through the point, i.e.,

$$\psi(x,t) = \frac{1}{2\pi} \int B_{\mathrm{pol}} dS.$$

A control-orientated model of the magnetic flux

$$\frac{\partial \psi(x,t)}{\partial t} = \frac{\eta_{\|}(x)}{\mu_0 a_{\mathrm{e}}^2} \left(\frac{\partial^2 \psi(x,t)}{\partial x^2} + \frac{1}{x} \frac{\partial \psi(x,t)}{\partial x} \right) + \eta_{\|}(x) R_0 j_{\mathrm{ni}}(x,t),$$

where $j_{\mathrm{ni}}(x,t) = j_{\mathrm{bc}}(x) + j_{\mathrm{nbi}}(x) + j_{\mathrm{eccd}}(x,t)$ with the boundary conditions

$$\frac{\partial \psi(0,t)}{\partial x} = 0 \qquad \frac{\partial \psi(1,t)}{\partial x} = -\frac{R_0 \mu_0 I_p}{2\pi}$$

Discretization distributed model

Due to the fact that the parameters $\eta_{\|}(x)$, $j_{\mathrm{bc}}(x)$, and $j_{\mathrm{nbi}}(x)$ are space dependent parameters, the PDE model has to be discretized in order to evaluate the magnetic flux $\psi(x,t)$.

$$\frac{\mathrm{d}\psi(x_i,t)}{\mathrm{dt}} = \frac{\eta_{\|_{i,j}}}{\mu_0 a_e^2}\left(c_1(i)\psi_{i+1} - c_2(i)\psi_i + c_3(i)\psi_{i-1}\right) + \eta_{\|_{i,j}} R_0 \left(j_{\mathrm{bc}}(x_i) + j_{\mathrm{nbi}}(x_i) + j_{\mathrm{eccd}}(x_i,t)\right)$$

with the discretization coefficients

$$c_1(i) = 1/2\,\frac{2\,x_i + \delta x}{\delta x^2 x_i}$$

$$c_2(i) = 2\,\frac{1}{\delta x^2}$$

$$c_3(i) = 1/2\,\frac{2\,x_i + \delta x}{\delta x^2 x_i}$$

State-space formulation spatially discretized model

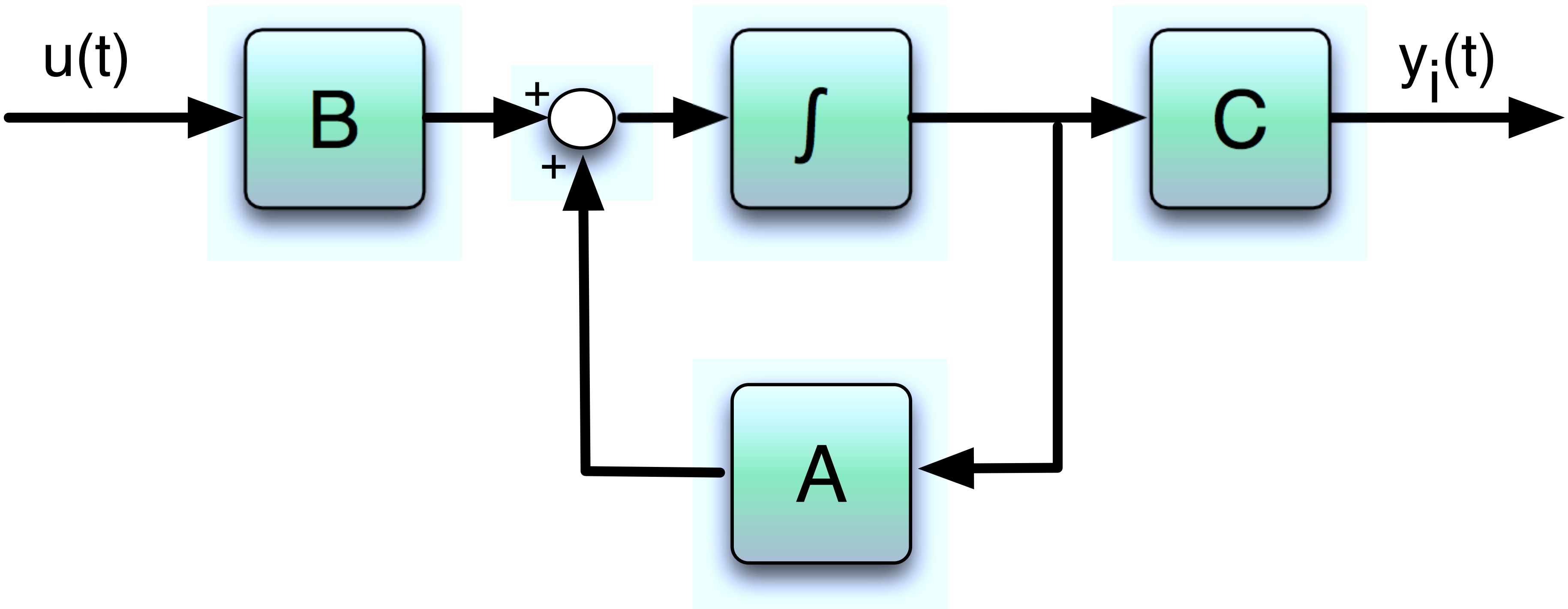
The state-space form of the spatially discretized model

$$\frac{\mathrm{d}\psi_\mathrm{i}(\mathrm{t})}{\mathrm{dt}} = \mathbf{A}\psi_i(t) + \mathbf{B}u_i(t)$$
$$y_i(t) = \mathbf{C}\psi_i(t)$$

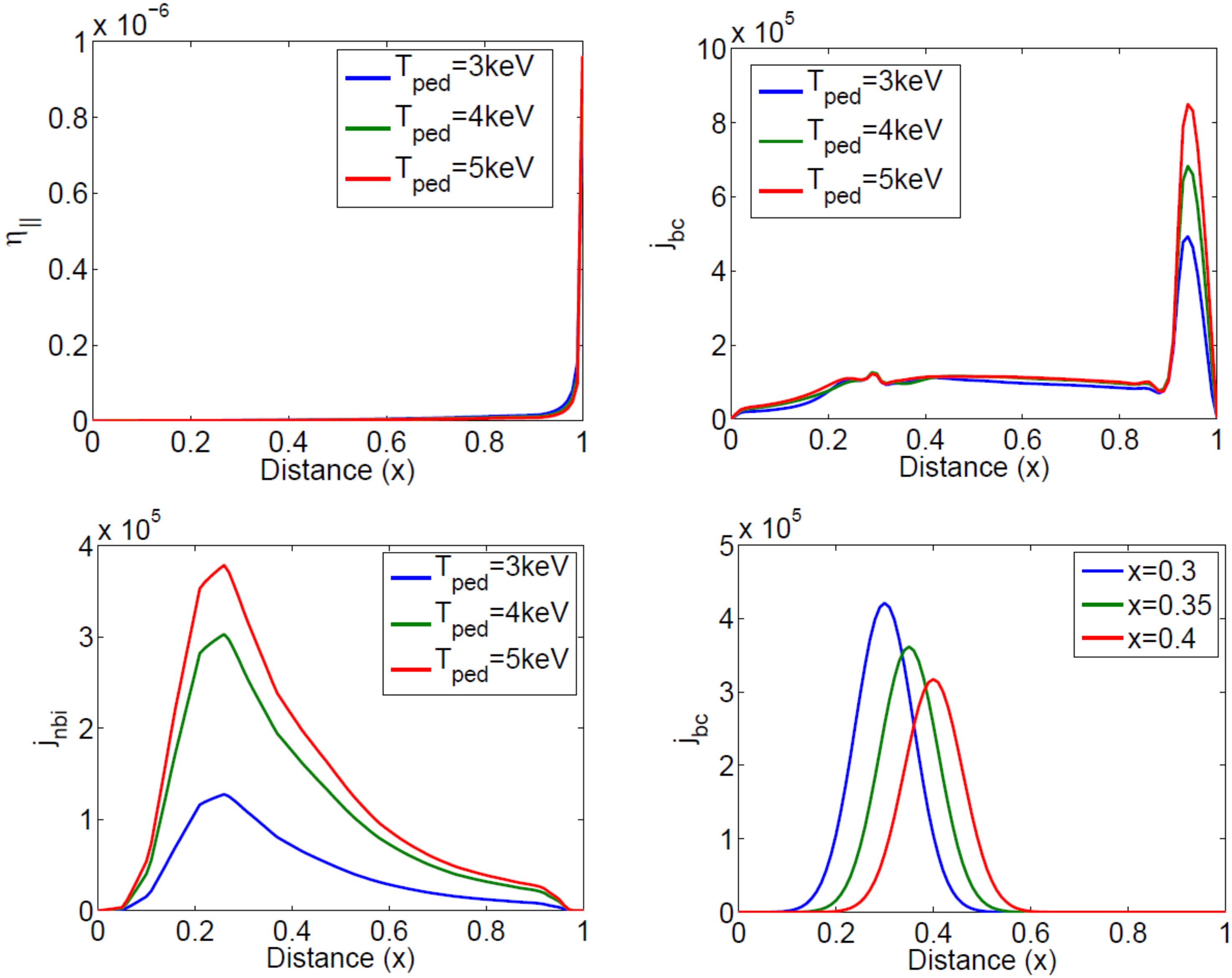
$\psi_i(t)$:	state vector	$\mathbb{R}^N$
$y_i(t)$:	output vector	$\mathbb{R}^M$
$u_i(\mathrm{t})$:	input vector	$\mathbb{R}^R$
$\mathbf{A}$:	system matrix	$\mathbb{R}^{N\times N}$
$\mathbf{B}$:	input matrix	$\mathbb{R}^{N\times R}$
$\mathbf{C}$:	output matrix	$\mathbb{R}^{M\times N}$

Transfer function: $\mathbf{G}(s) = \dfrac{y_i(s)}{u_i(s)} = \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B}$

Graphical representation of State-Space model



Input to state-space model from CRONOS



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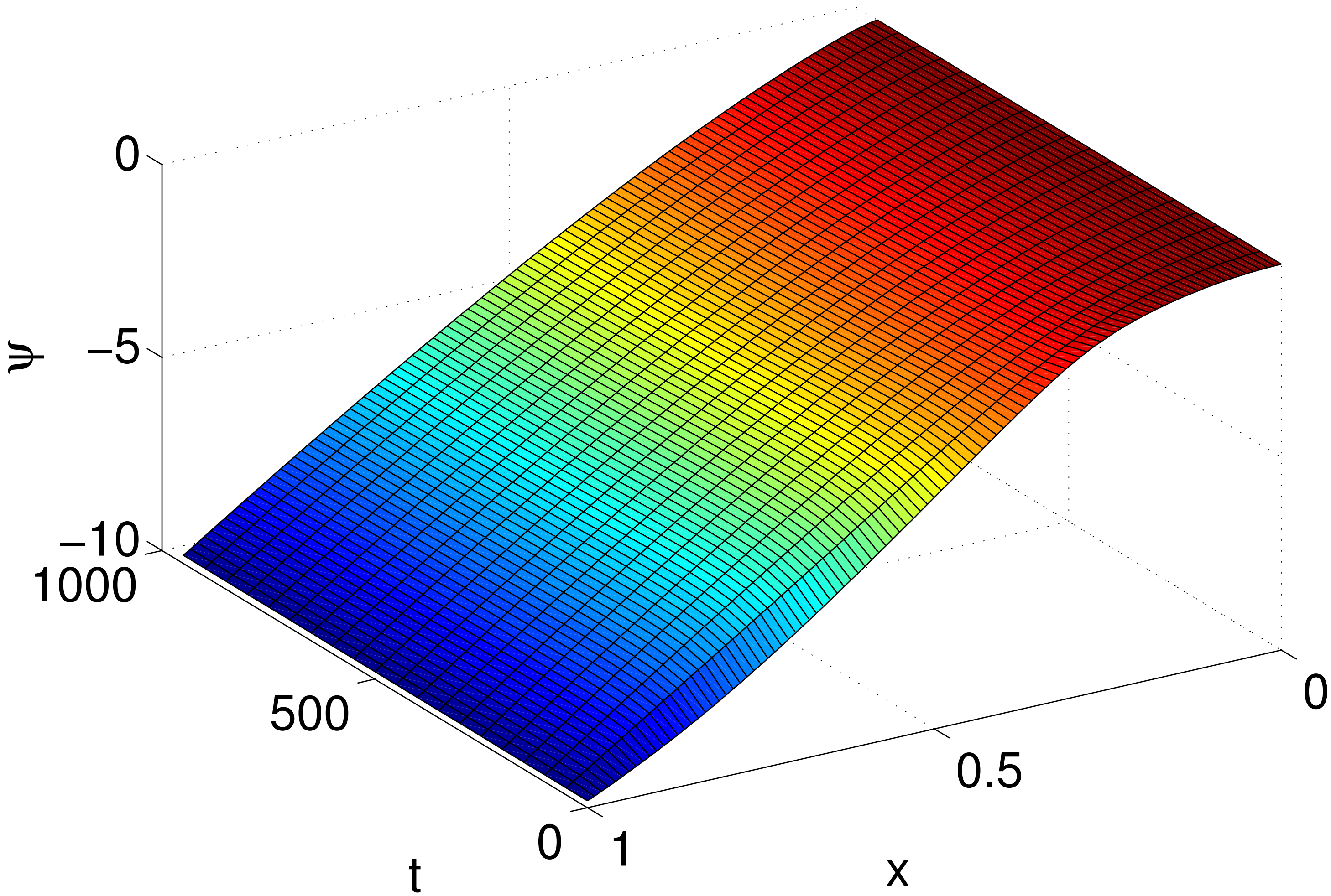
Variation poloidal flux with state-space model



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Controllability analysis

The controllability matrix $\mathcal{C}_i$ equals

$$\mathcal{C}_i(\mathbf{A}, \mathbf{B}) = \begin{bmatrix} \mathbf{B} & \mathbf{AB} & \mathbf{A}^2\mathbf{B} & \dots & \mathbf{A}^{i-1}\mathbf{B} \end{bmatrix}$$

The observability matrix $\mathcal{O}_i(\mathbf{A}, \mathbf{C})$

$$\mathcal{O}_i(\mathbf{A}, \mathbf{C}) = \begin{bmatrix} \mathbf{C} \\ \mathbf{CA} \\ \mathbf{CA}^2 \\ \vdots \\ \mathbf{CA}^{i-1} \end{bmatrix}$$

The system is controllable/observable if the controllability/observability matrix has full rank.

$$\mathrm{rank}(\mathcal{C}_i) = \mathrm{N} \qquad \mathrm{rank}(\mathcal{O}_i) = \mathrm{N}$$

Controllability / observability Gramian and reachable sets

For large scale systems, where $N > 10$, there are elements that require a significant amount of energy in terms of states.
The controllability/observability Gramian

$$\mathcal{P} = \mathcal{C}_\infty(\mathbf{A}, \mathbf{B})\mathcal{C}_\infty^T(\mathbf{A}, \mathbf{B}) = \sum_{i=0}^{\infty} \mathbf{A}^i \mathbf{B}\mathbf{B}^T (\mathbf{A}^T)^i,$$

$$\mathcal{Q} = \mathcal{O}_\infty(\mathbf{C}, \mathbf{A}))_\infty^T(\mathbf{C}, \mathbf{A}) = \sum_{i=0}^{\infty} (\mathbf{A}^T)^i \mathbf{C}^T \mathbf{C}\mathbf{A}^i$$

The reachable sets from the given initial condition

$$J_{\mathrm{con}}(\psi_i) = \psi_i^T \mathcal{P}^{-1}\psi_i \qquad J_{\mathrm{obs}}(\psi_i) = \psi_i^T \mathcal{Q}\psi_i$$

Singular values for accessible states

In general, a state coordinate transformation produces an equivalent model in another coordinate system in which $\overline{\psi_i}(t) = \mathbf{T}\psi_i(t)$

$$\frac{\mathrm{d}\overline{\psi_i}(t)}{\mathrm{d}t} = \mathbf{TAT}^{-1}\overline{\psi_i}(t) + \mathbf{TB}u_i,$$
$$y_i = \mathbf{CT}^{-1}\overline{\psi_i}(t),$$

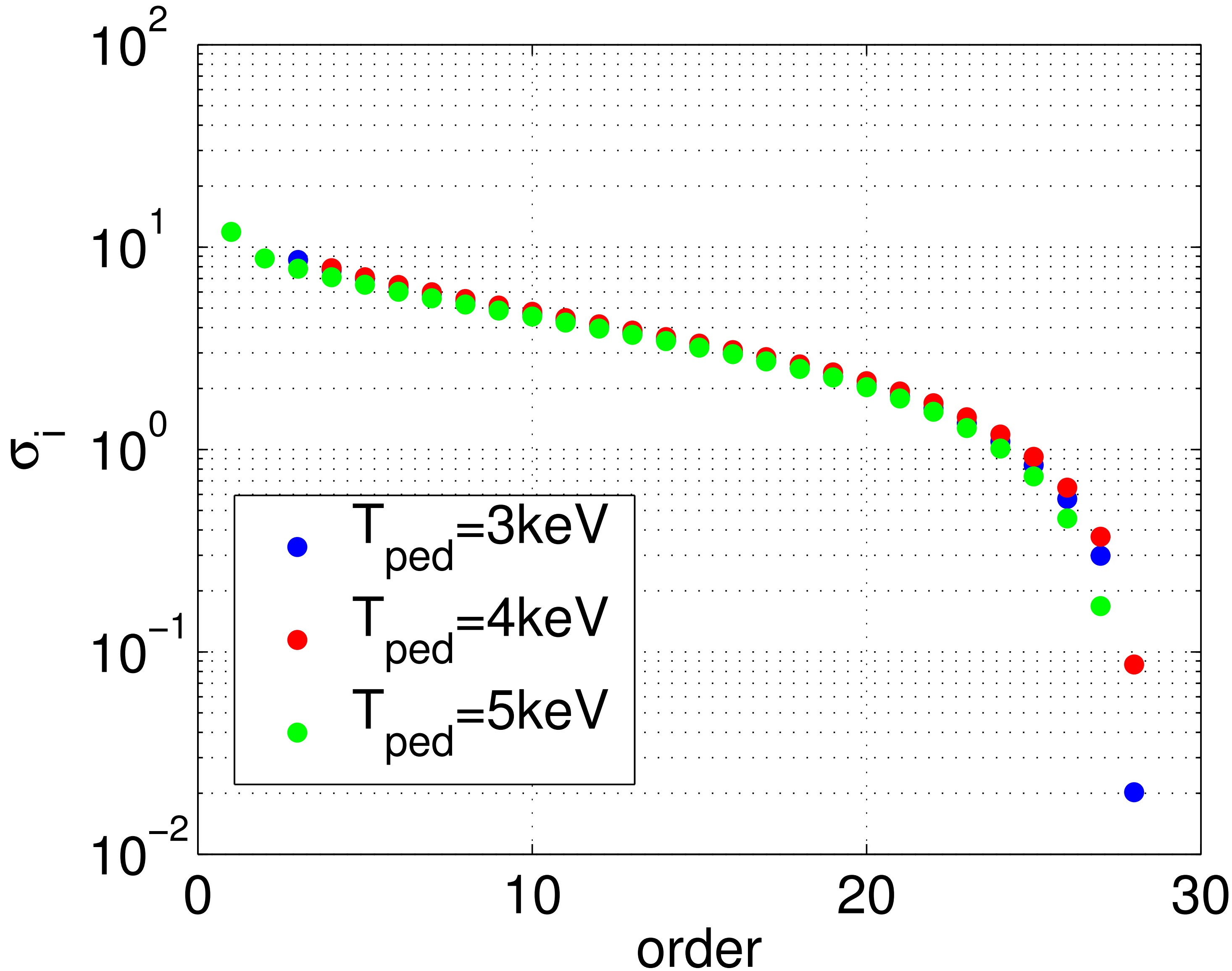
The associated Gramians $\overline{\mathcal{P}}$ and $\overline{\mathcal{Q}}$

$$\overline{\mathcal{P}} = \mathbf{T}\mathcal{P}\mathbf{T}^T \quad \overline{\mathcal{Q}} = \mathbf{T}^{-T}\mathcal{Q}\mathbf{T}^{-1} \qquad \overline{\mathcal{P}\mathcal{Q}} = \mathbf{T}\mathcal{P}\mathcal{Q}\mathbf{T}^{-1}$$

$$\overline{\mathcal{P}} = \overline{\mathcal{Q}} = \mathrm{diag}(\sigma_1, \sigma_2 ..., \sigma_\mathrm{N})$$

$$\sigma_i = \sqrt{\lambda_i(\mathcal{P}\mathcal{Q})} = \sqrt{\lambda_i(\overline{\mathcal{P}}\overline{\mathcal{Q}})}, \qquad i = 1, 2, ..., N$$

Singular values



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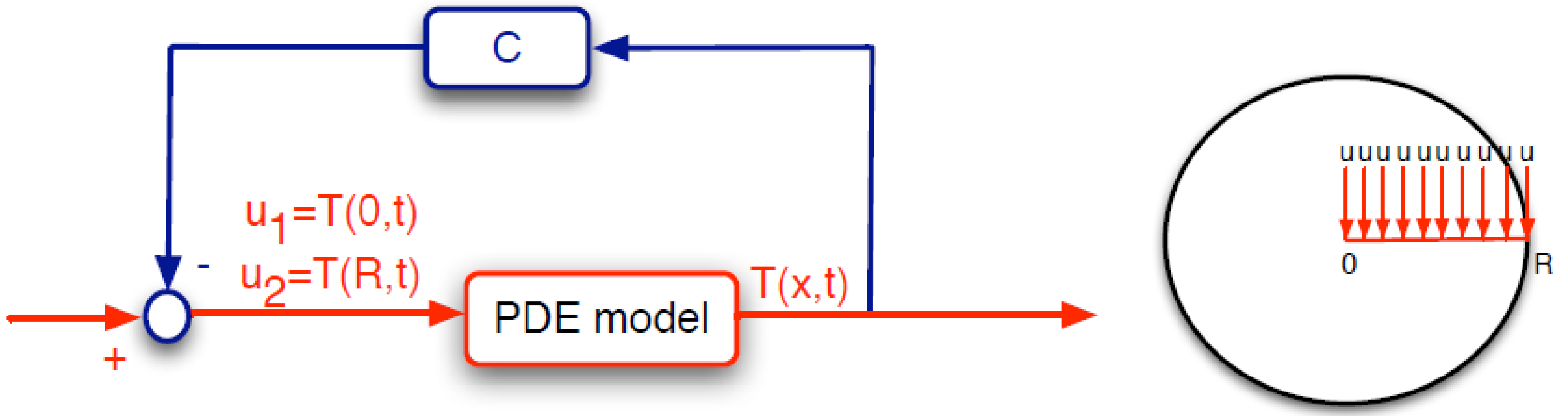
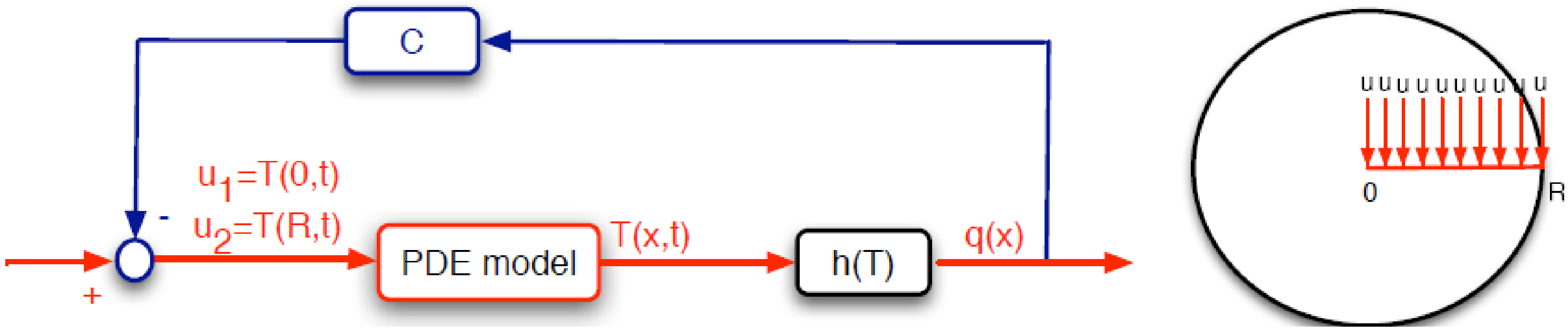
Control


C
u1=T(0,t)
u2=T(R,t)
-
+
PDE model
T(x,t)
u u u u u u u u u u
0
R


C
u1=T(0,t)
u2=T(R,t)
-
+
PDE model
T(x,t)
h(T)
q(x)
u u u u u u u u u u
0
R

Summary

• Energy confinement in tokamaks limited by turbulence driven by temperature gradients

• Turbulence onset characterized by critical gradients, predicted by linear theory

• Critical gradients predicted to be sensitive to the current profile

•Discretized distributed state-space model for the flux distribution in ITER-Hybrids derived

•Observability / Controlability analysis carried out for ITER Hybrid scenarios, using detailed input from Cronos simulations

•Simulation suggests that only a subset of the states is effectively accessible

•Model based closed-loop control optimization remains to be carried out (before October 2011) based on <s/q> cost function. Expected to yield overall improved q and s-profiles (Also maximization of low s volume)

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