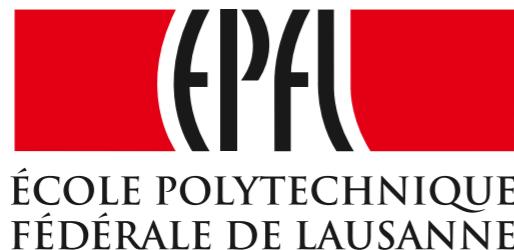
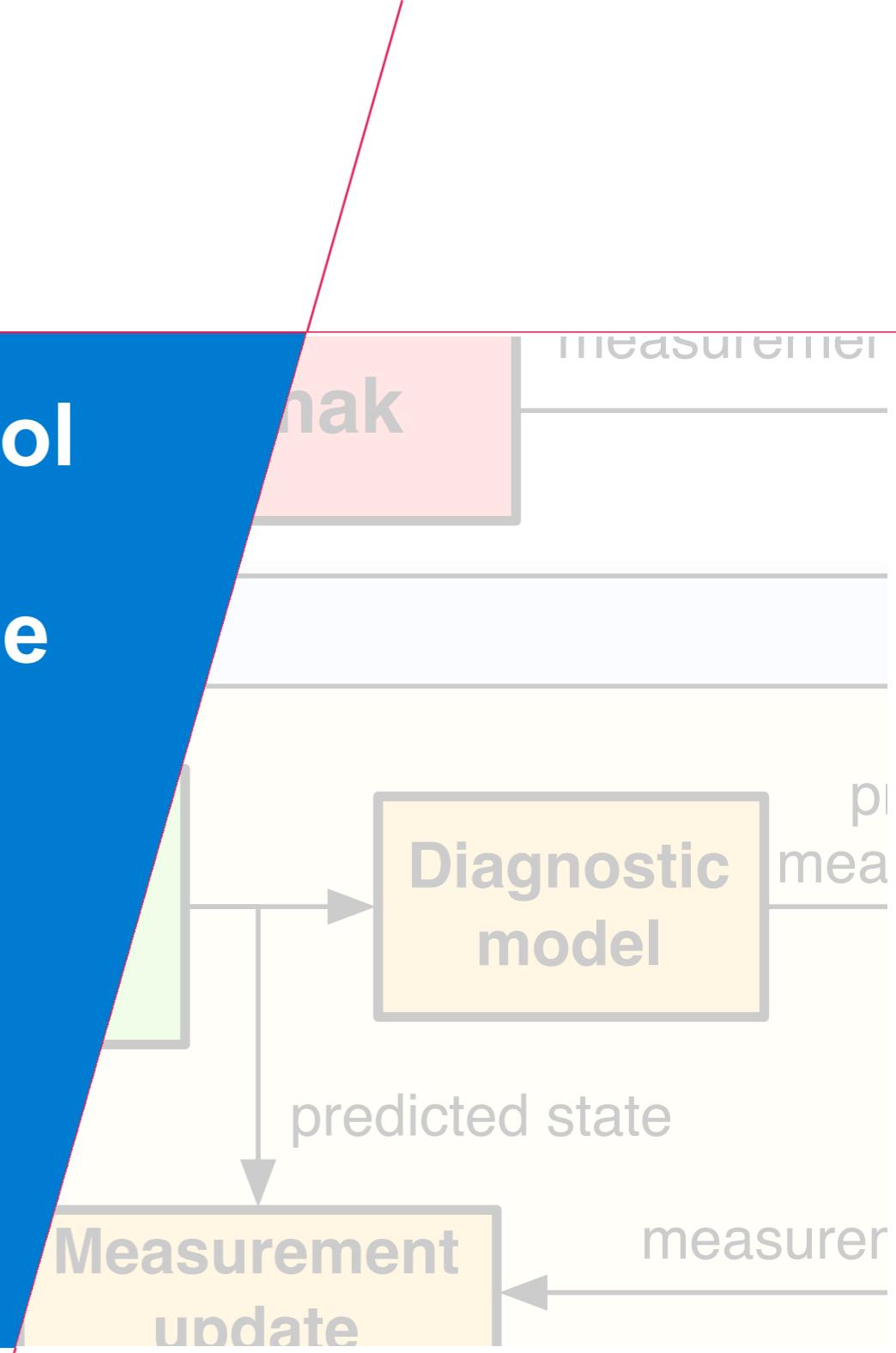


# Real-time reconstruction, control and optimization of plasma profiles using the RAPTOR code

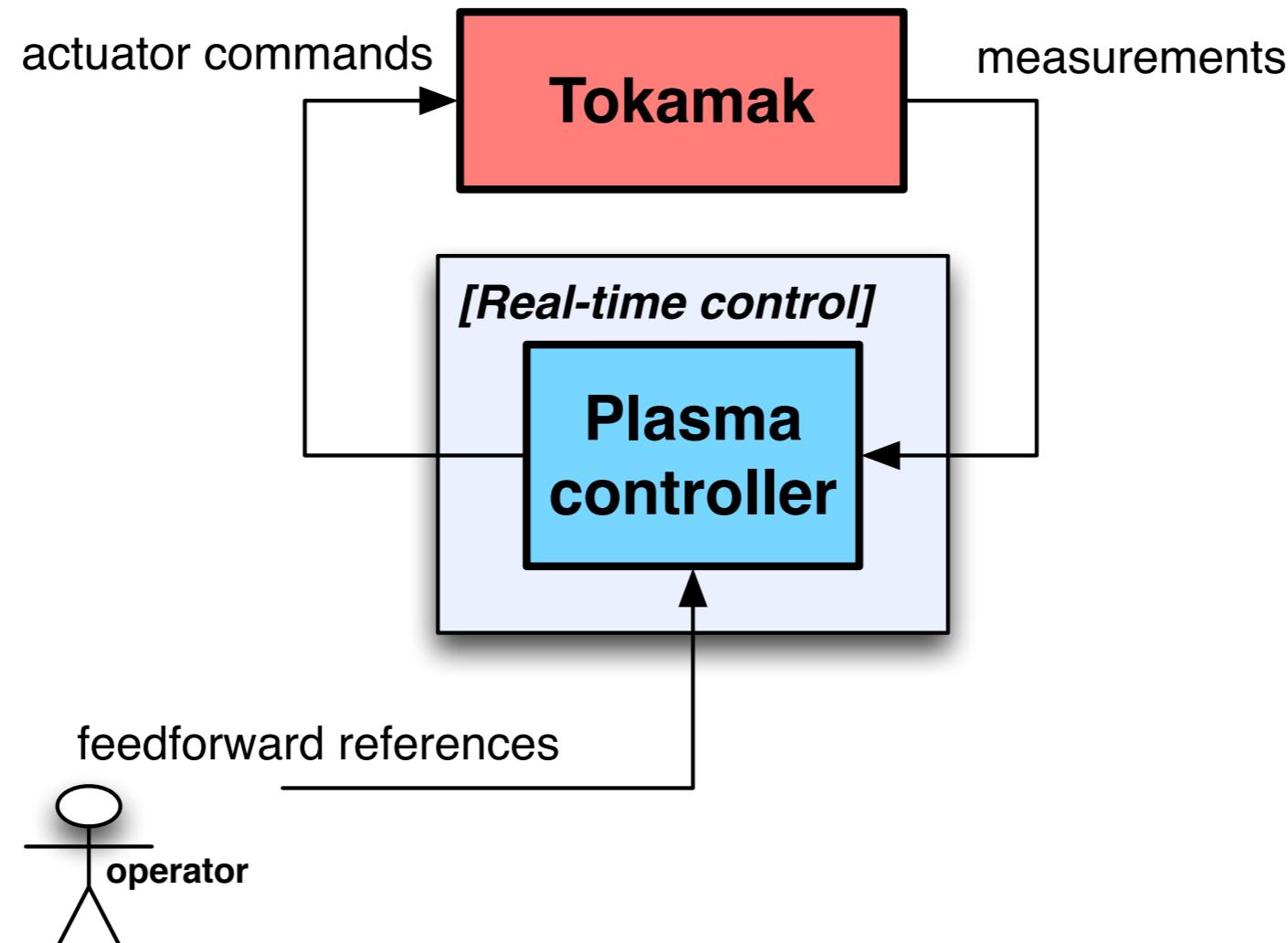
Federico Felici

Eindhoven University of Technology (The Netherlands)  
Department of Mechanical Engineering  
Control Systems Technology Group

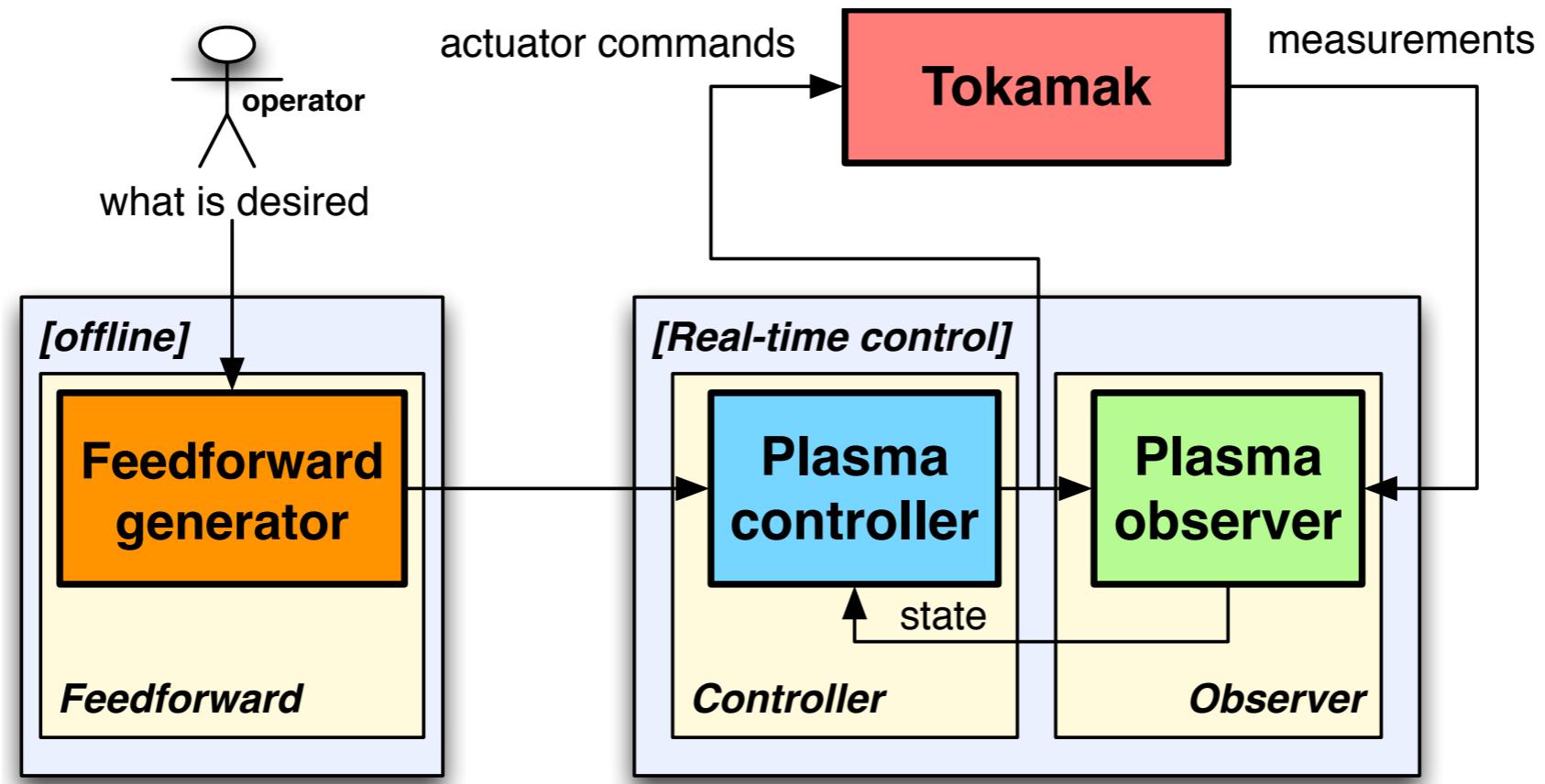


Technische Universiteit  
**Eindhoven**  
University of Technology

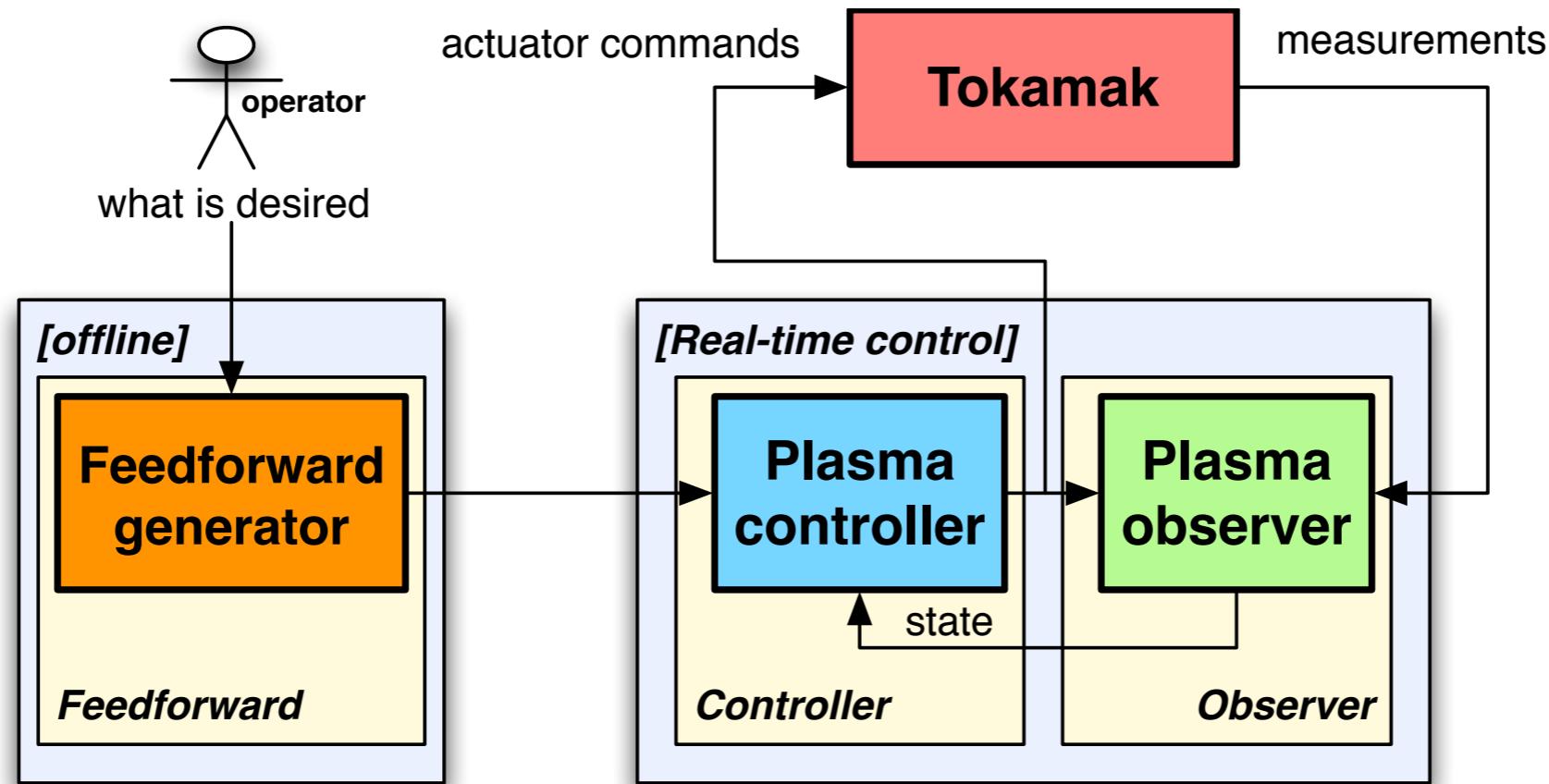
# Most control loops as used in tokamaks today



# Model-based control system: components

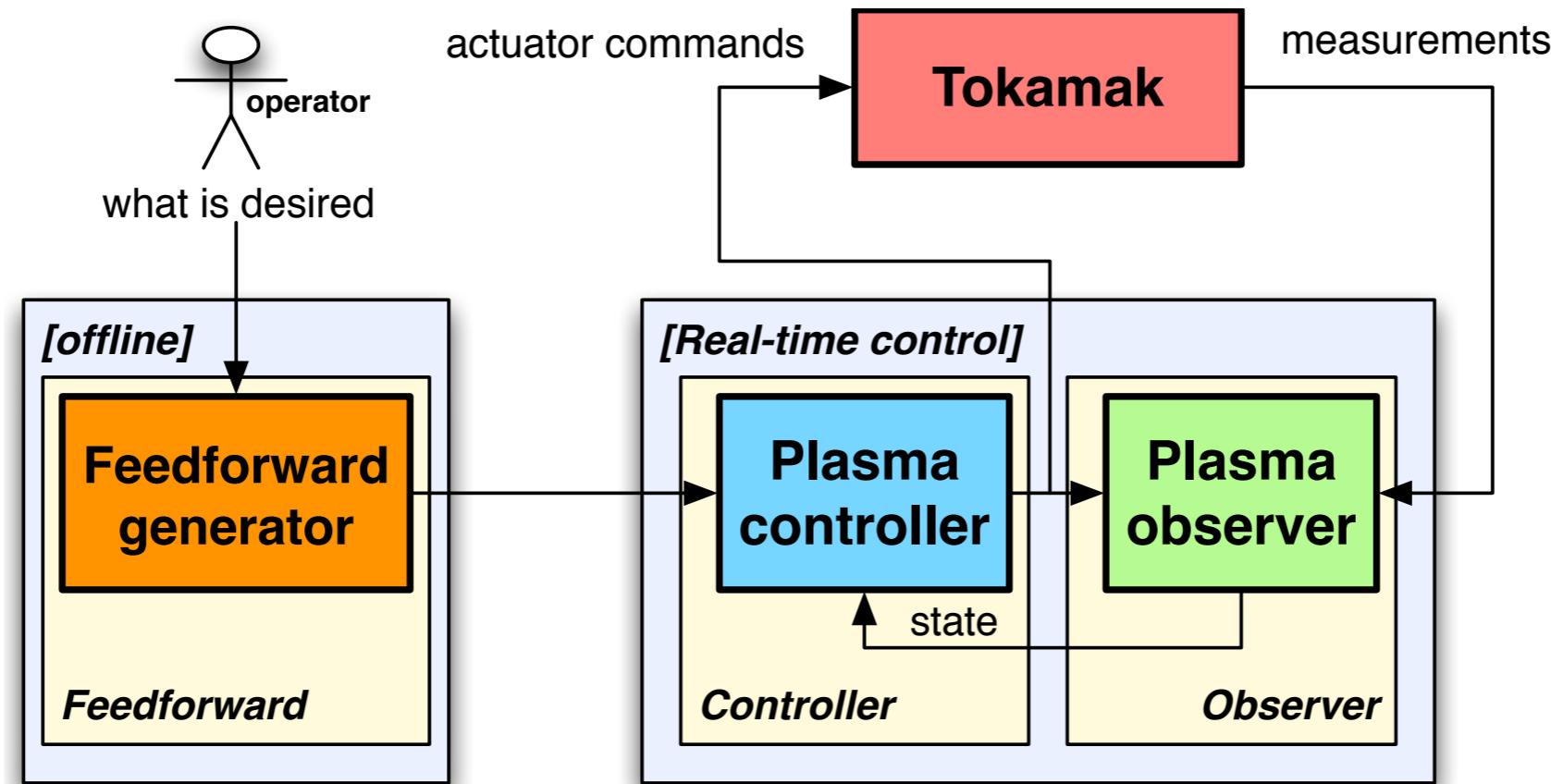


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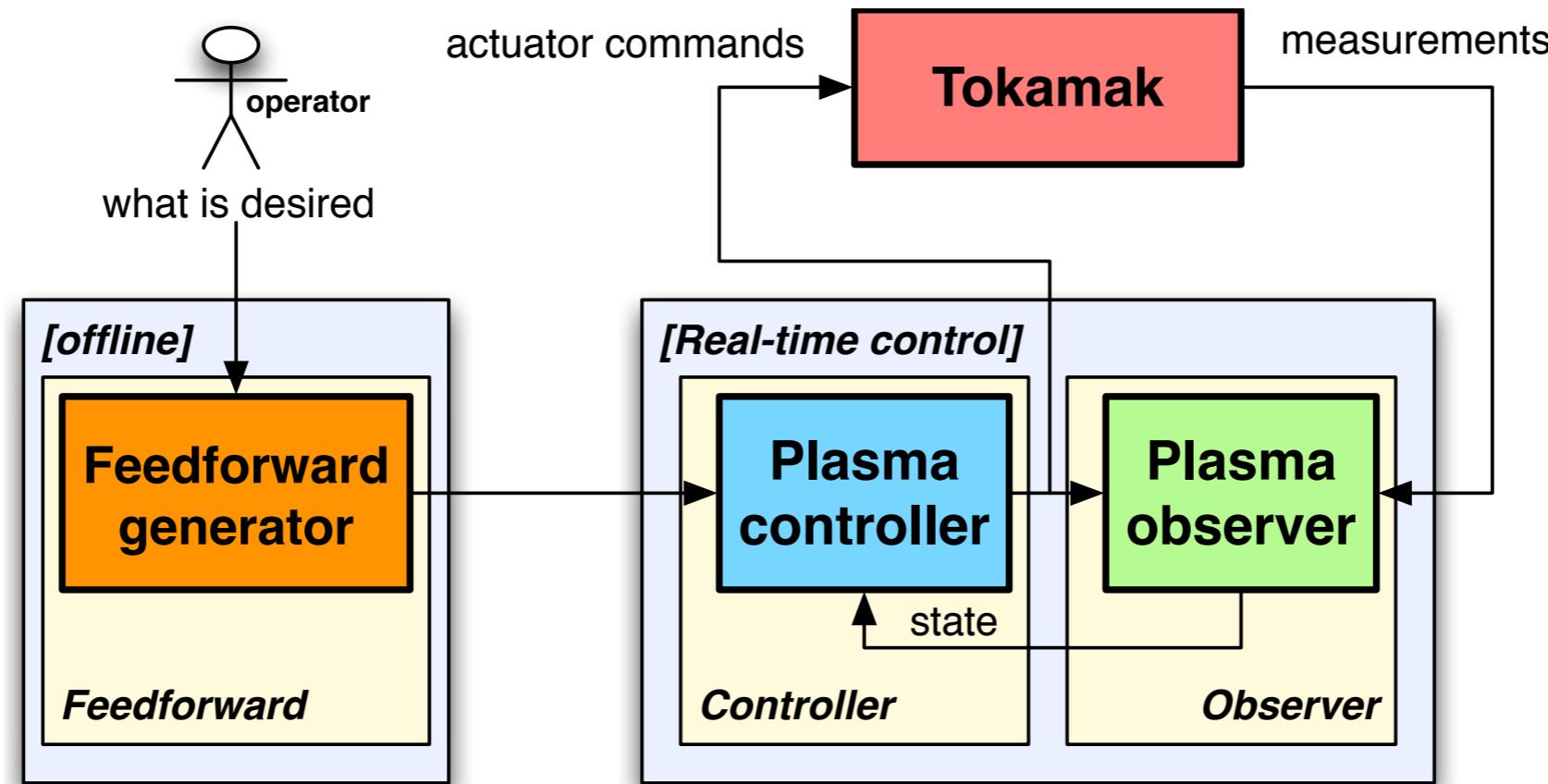
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  - Layer of abstraction for operators

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- State estimation (observer) separate from state control
  - Appropriate when measurements are noisy and/or incomplete
- Automatic generation of feedforward trajectories
  - Layer of abstraction for operators
- Model-based plasma controller
  - Use model to predict the future and determine best control strategy

# Models for model-based control

# Models for model-based control

- Use first-principle models deeply embedded in design and implementation of real-time control
  - What models? not full physics models, but *control-oriented* models.
    - Capability to run in real-time (or faster)
    - Capture main dynamics and coupling, but no perfection needed

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- This talk: Presentation of new control-oriented code  
**RAPTOR (Rapid Plasma Transport Simulator)**
  - Features
  - Applications
    - *Fast simulator for rapid scenario development, controller design, ...*
    - Profile reconstruction
    - Trajectory optimization
    - Real-time feedback control and prediction

# RAPTOR code contains key nonlinear couplings affecting the dynamics of profile evolution

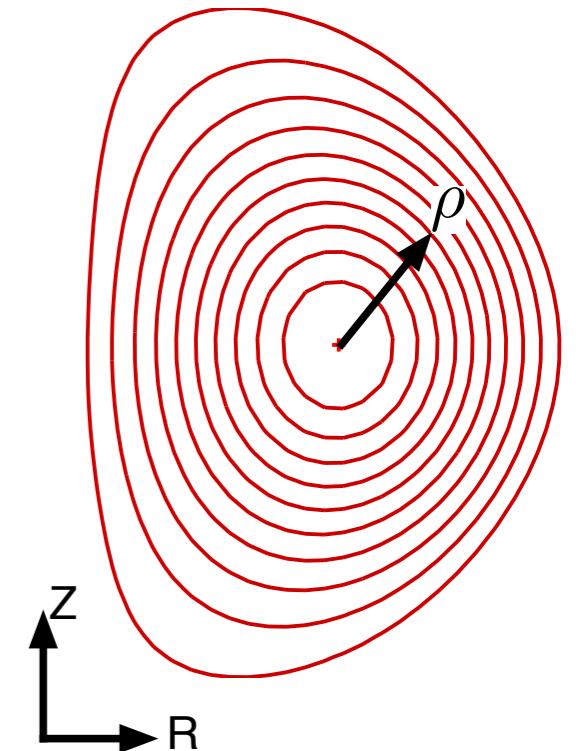
- Noncircular, axisymmetric, fixed poloidal flux surface shape
- 1D, (flux surface averaged) poloidal flux diffusion

$$\sigma_{||} \frac{\partial \psi}{\partial t} = \frac{R_0 J^2}{\mu_0 \rho} \frac{\partial}{\partial \rho} \left( \frac{G_2}{J} \frac{\partial \psi}{\partial \rho} \right) - \frac{V'}{2\pi\rho} (j_{BS} + j_{ext})$$

- Neoclassical conductivity & bootstrap : Sauter-Angioni
- Electron temperature diffusion

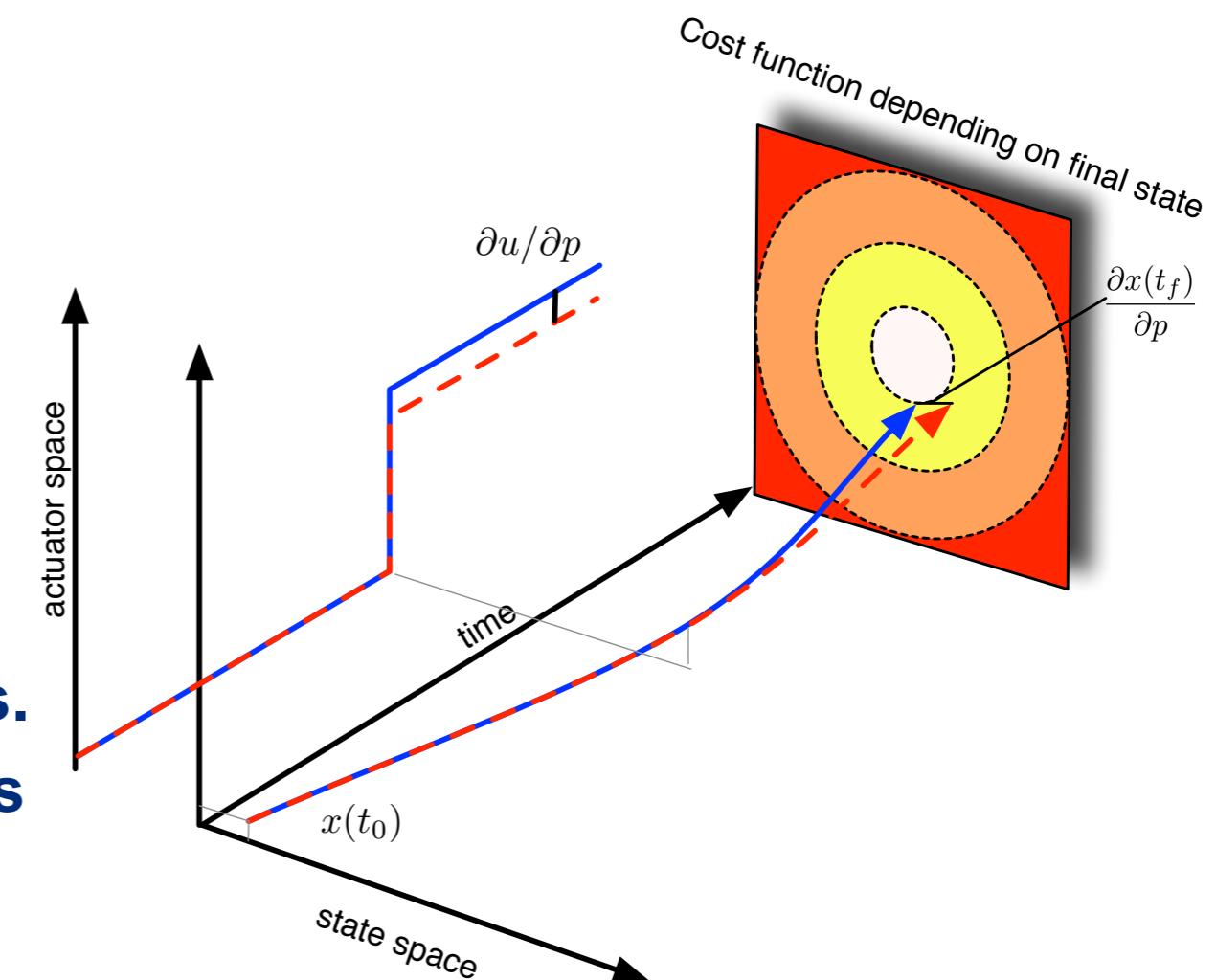
$$V' \frac{\partial}{\partial t} [n_e T_e] = \frac{\partial}{\partial \rho} G_1 V' n_e \chi_e \frac{\partial T_e}{\partial \rho} + V' P_e$$

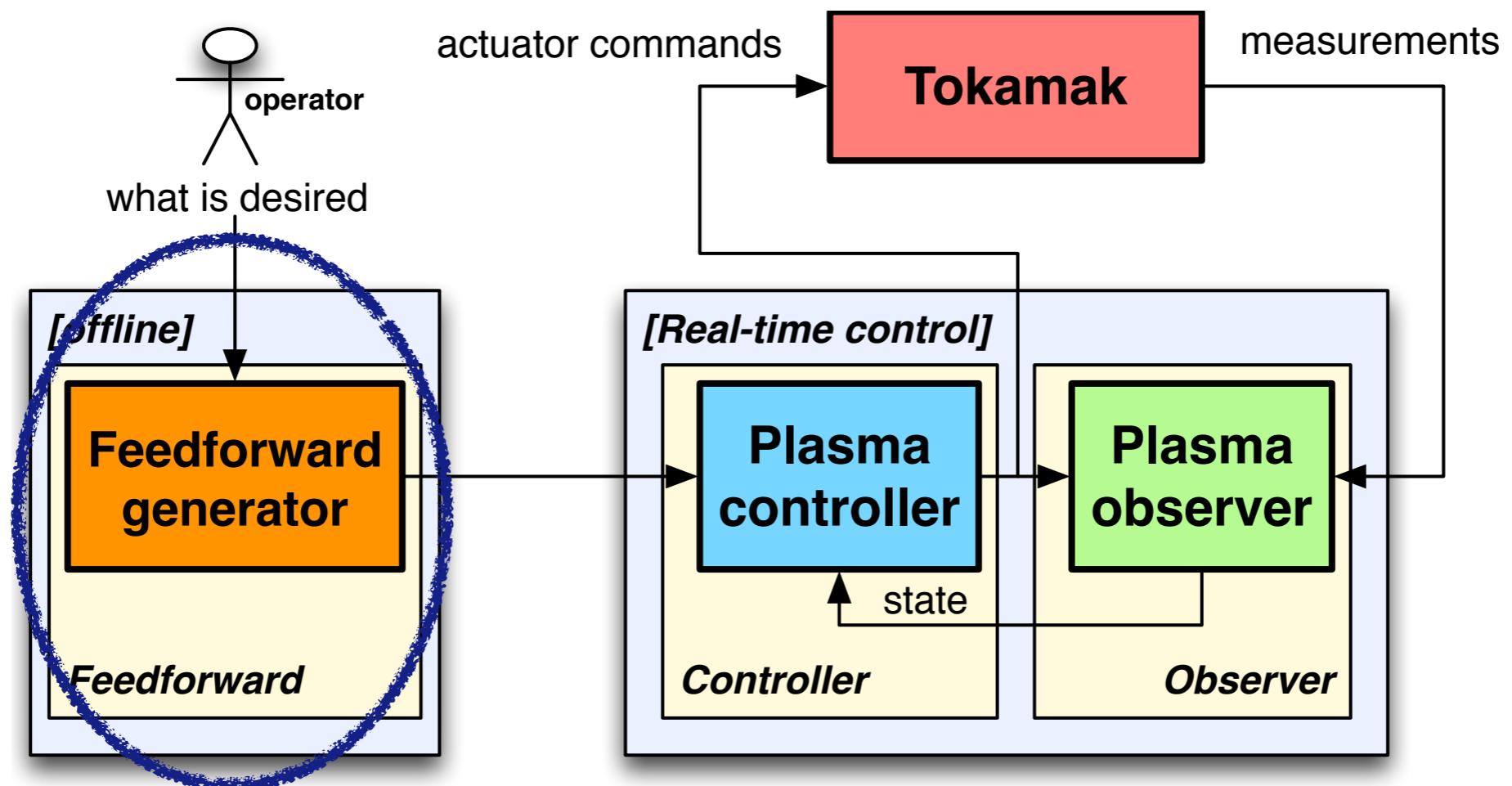
- Prescribed density profile evolution,  $T_i = k^* T_e$
- Ad-hoc analytical model for thermal diffusivity
- Sources
  - Parametrized model for EC deposition
  - Pencil beam model for NBI (P. Geelen)
  - Alpha particle, radiation, brehmsstr. included (J. van Dongen)



# RAPTOR uses implicit solver which calculates Jacobians at all times, gives local linearization

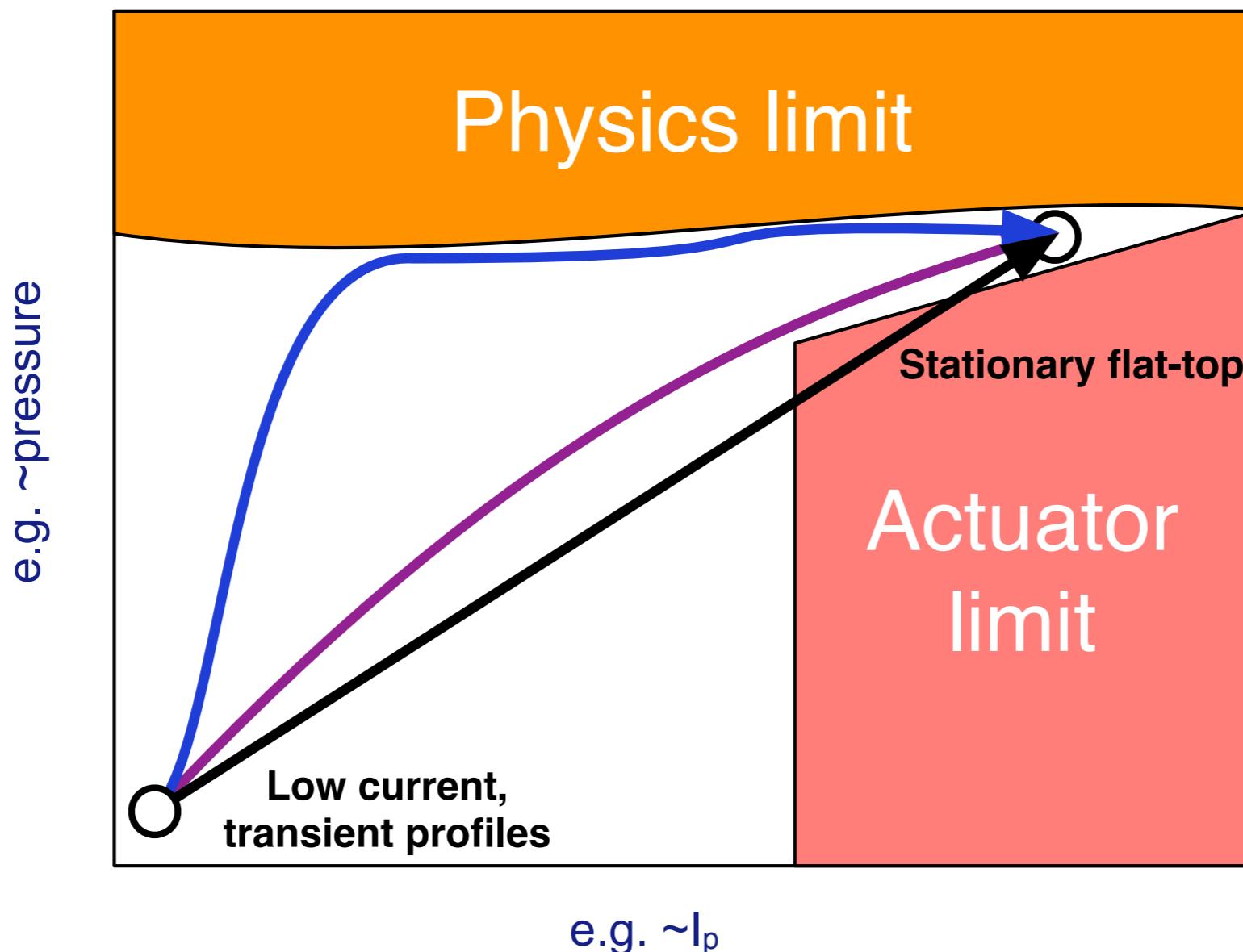
- Numerics:
  - Cubic spline finite elements
  - Fully implicit, full Newton steps
  - Analytic Jacobians for all terms
  - few ms per time step
- Gradients computed using forwards sensitivity method
  - State sensitivities:  $dx/dp$  at all times.
  - Linearization of the profile dynamics around the profile trajectory - local linear model
  - Important for numerical optimization and controller design
- Model parameter optimization
  - Automatically based on experimental data - new work by P. Geelen (to be submitted)





# Model-based optimization of open-loop actuator trajectories

Tokamak operational space  
Which route to take?



# Optimization problem: ingredients

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- **Cost function  $J$ : reflects desired properties of plasma**
  - **Weighted sum of several profile-dependent terms**
    - distance from target profiles ( $q$ ,  $T_e$ ,  $E_{\parallel}$ ...)
    - Flux consumption (for longer pulse)
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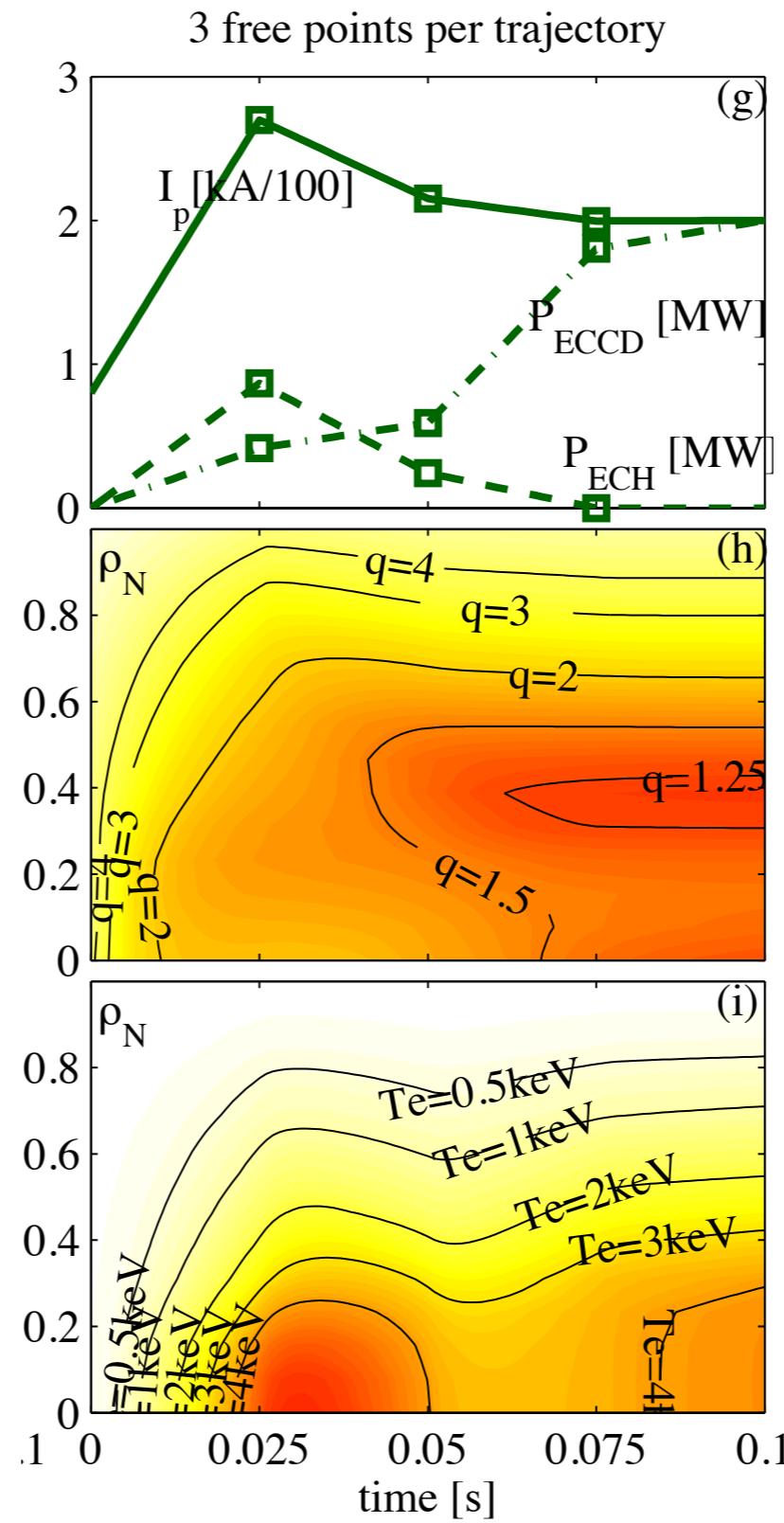
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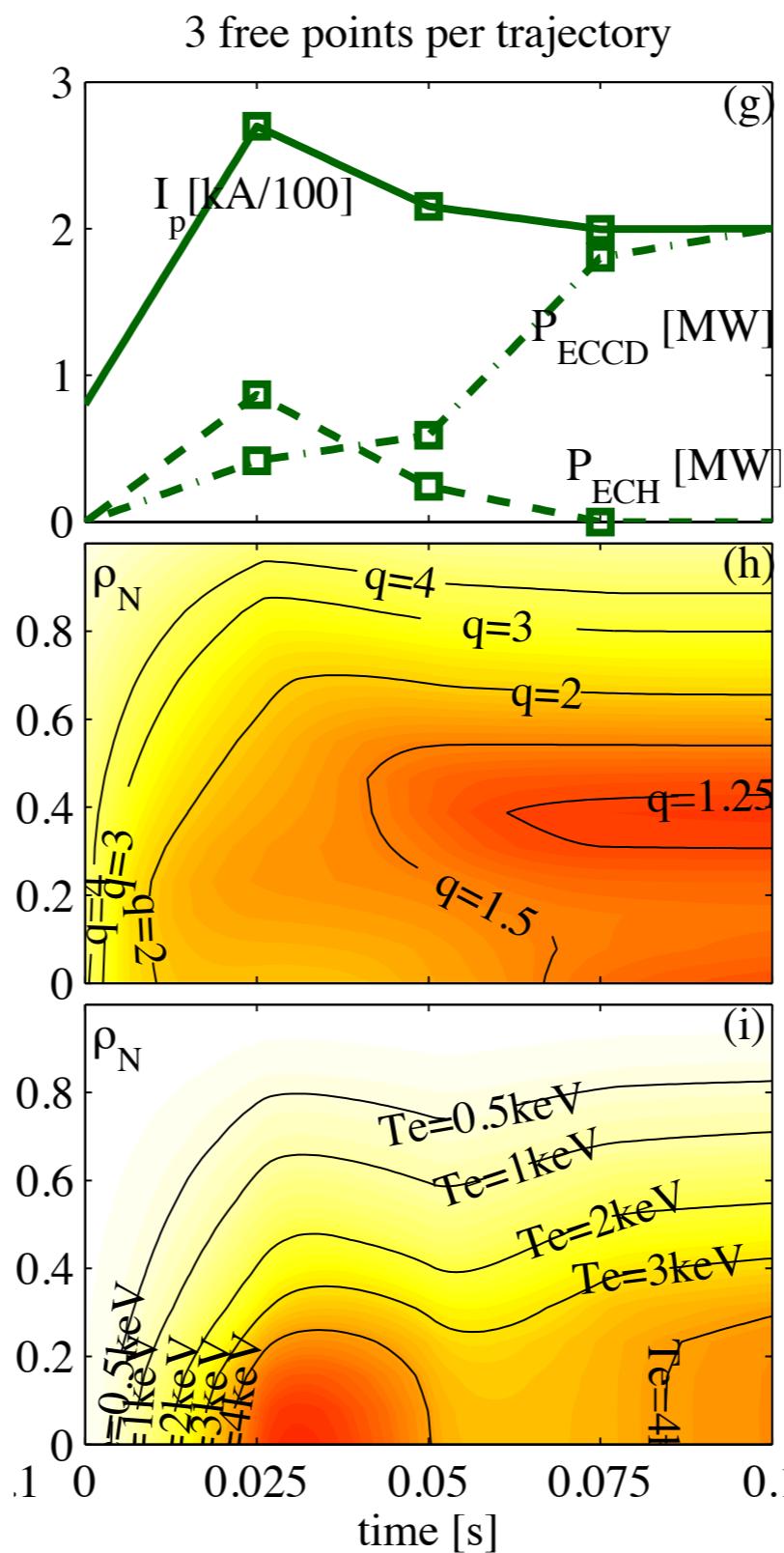
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- **Solution:** Sequential Quadratic Programming
  - Gradients  $dJ/dp$   $dC/dp$  are *known*, this greatly speeds up computations.

# Results for ramp up to ‘hybrid’ q profile show benefit of early heating and $I_p$ overshoot



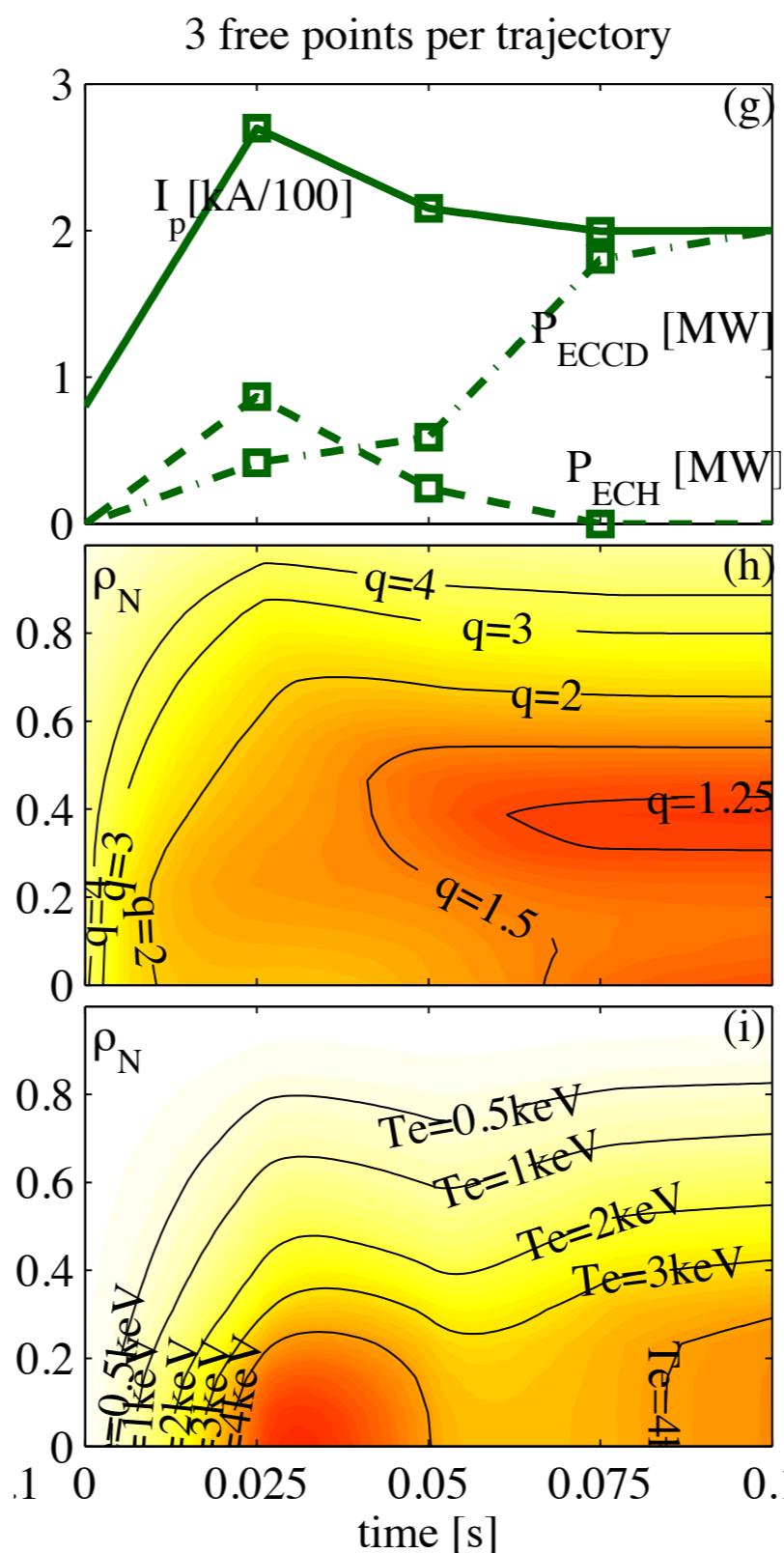
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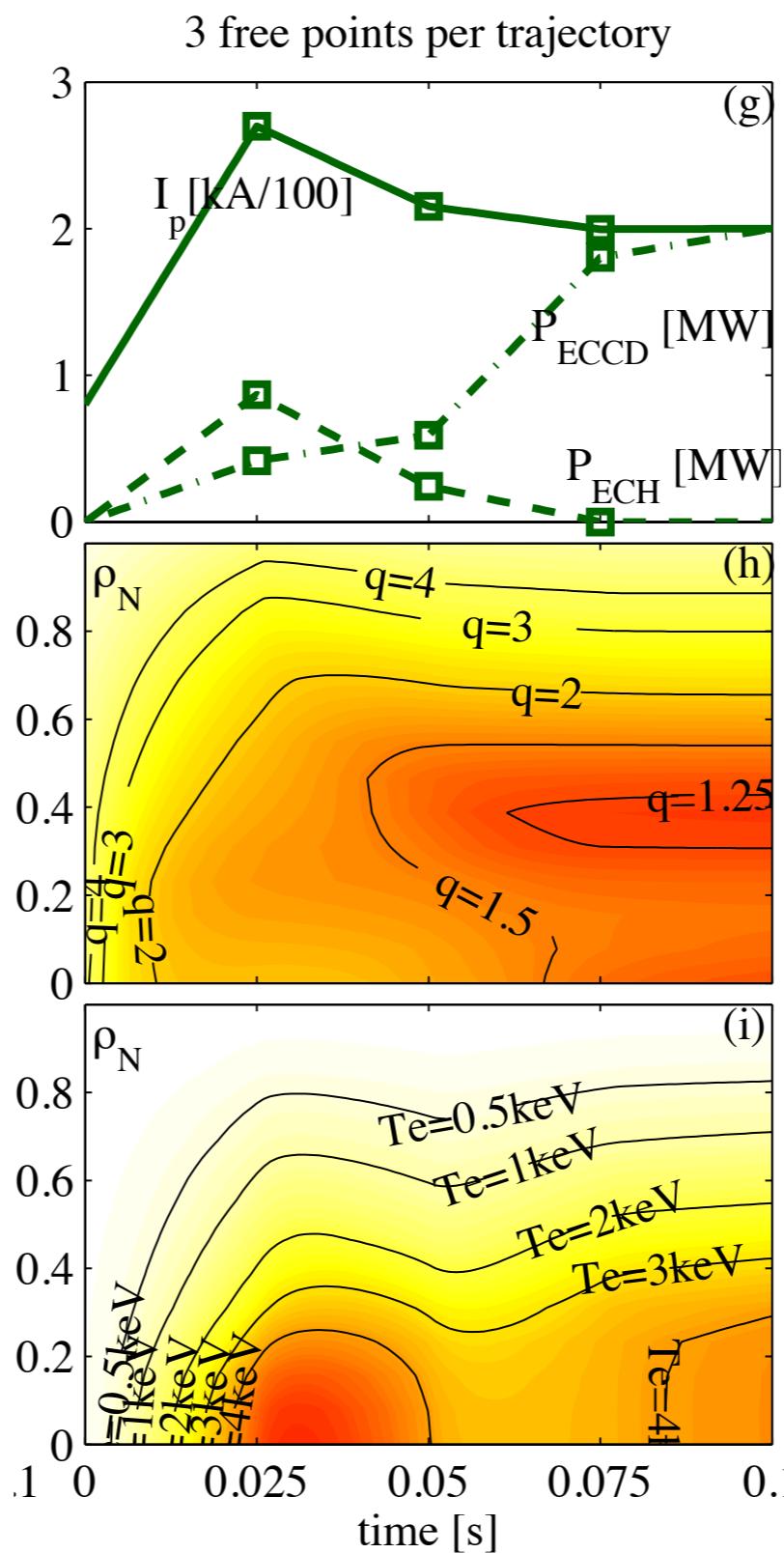
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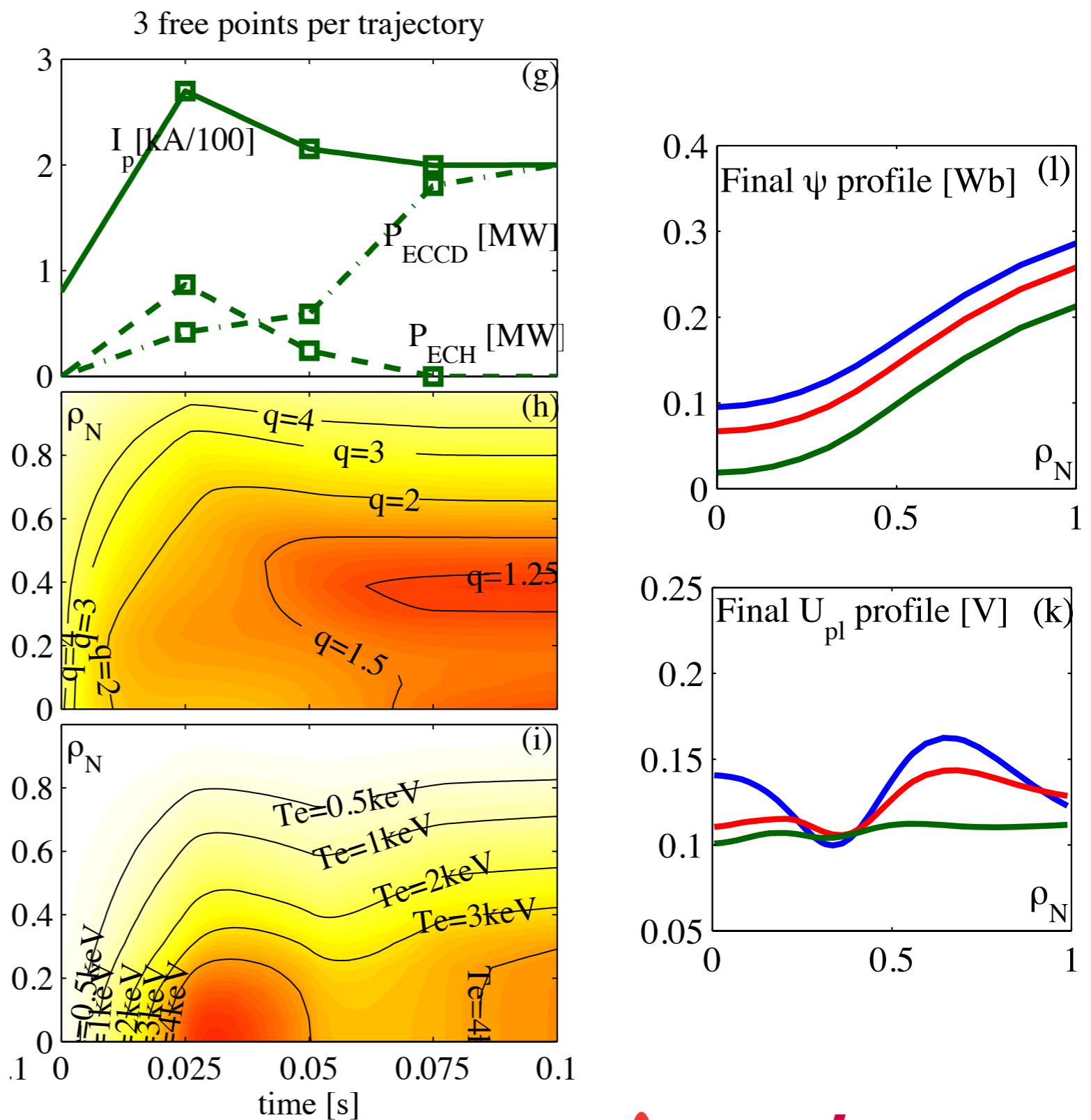
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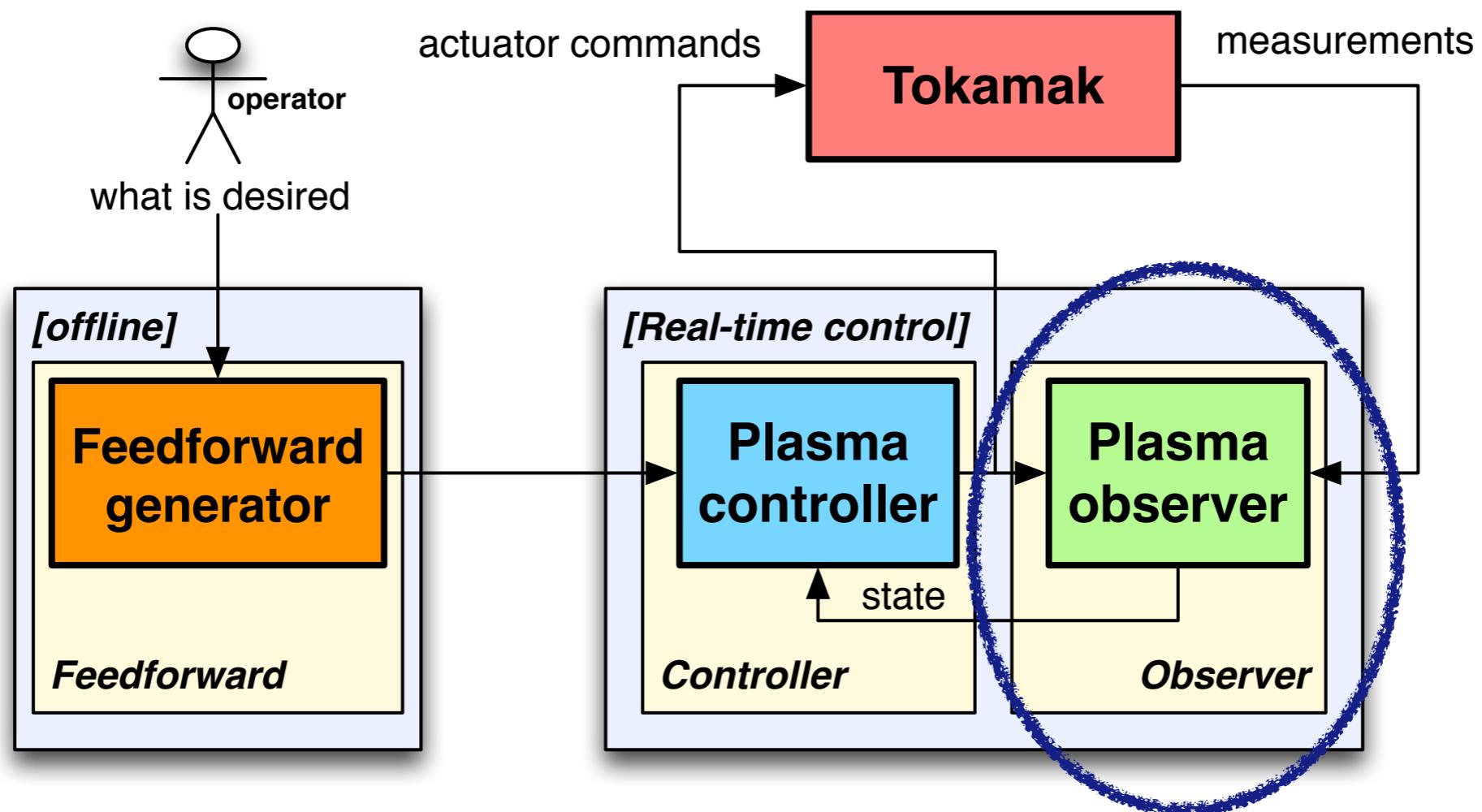
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  - Minimize flux cons.
- Multiple constraints
- Result:
  - Lower flux cons.
  - Flatter  $U_{pl}$  profile



# Perspectives and future plans for trajectory optimization

- Optimization of ramp-up
  - First applied to simulated TCV ramp-up [F. Felici PPCF 2012]
  - Recently applied to ITER hybrid scenario simulations  
(see next talk by J. van Dongen)
  - Validation of optimized ramp-up trajectories in existing tokamaks envisaged
- Optimization of ramp-down ?
  - Appropriate cost functions/constraints?
  - Optimal (varying?)  $I_p$  rampdown rate
  - Timing removal of different heating/cd actuators accounting for profile dynamics?
  - Need to take shape evolution into account ?
  - Provide guidelines to experiments and simulations: save valuable time



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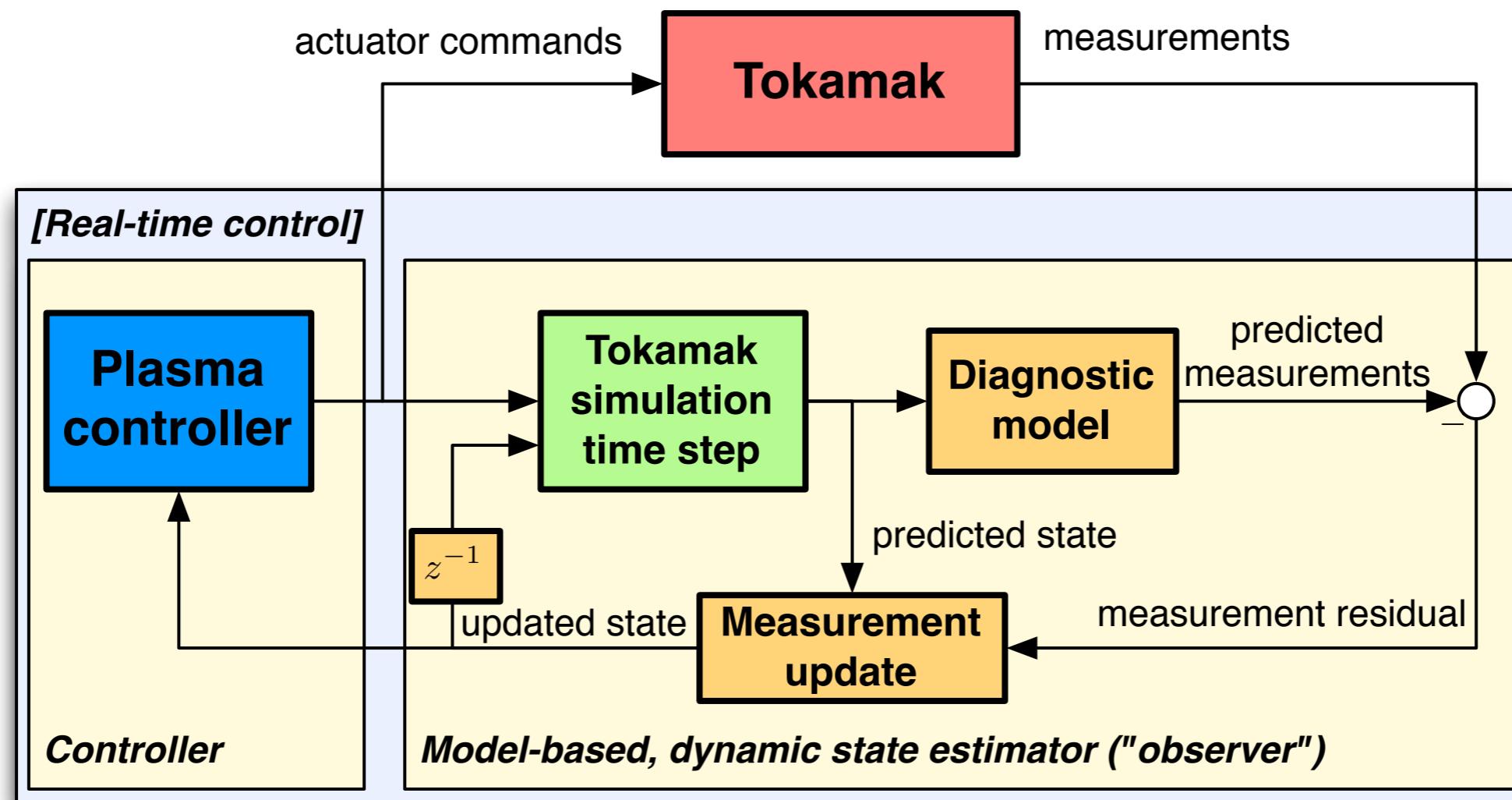
- In the past: feed measurements directly to plasma controller
- Today: constrained equilibrium reconstruction for some controlled quantities (e.g. shape,  $q$ ), direct feedback for others (e.g. density)
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  - But: we run post-shot interpretative transport simulations to analyze shots in detail, measurements often included in ad hoc fashion.
  - Model-based plasma state reconstruction, merge model prediction and diagnostic measurements
    - Amounts to performing a *real-time, measurement constrained simulation* of the plasma time evolution.
      - Known in control literature as *dynamic state observer*, or *Kalman filter*.
      - Widely used in robotics, image processing, broad literature exists

# Predict next plasma state with model, correct by diagnostic measurements

- Components of model-based state observer
  - Forward simulator (predict state one step ahead)
  - Diagnostic model (predict measurements from predicted state)
  - Measurement update (correct state based on actual measurements)



# State observer

- Full state knowledge means *everything*, not just what you measure.
  - $q$ , shear,  $T_e$ ,  $dT_e/drho$ ,  $j_{aux}$ ,  $j_{bs}$ ,  $j_{oh}$  profiles
  - Confinement time, non-inductive current fraction, H-factor, ...
- Measurement update law reflects confidence in models vs measurements
  - Diagnostic noise?
    - Filtered out naturally by model: accept only variations consistent with model time scales.
  - Disturbances / faults ?
    - Detect systematic disturbances of measured evolution w.r.t. model
    - Classify as normal (e.g. model mismatch) or off-normal (e.g. imminent disruption)

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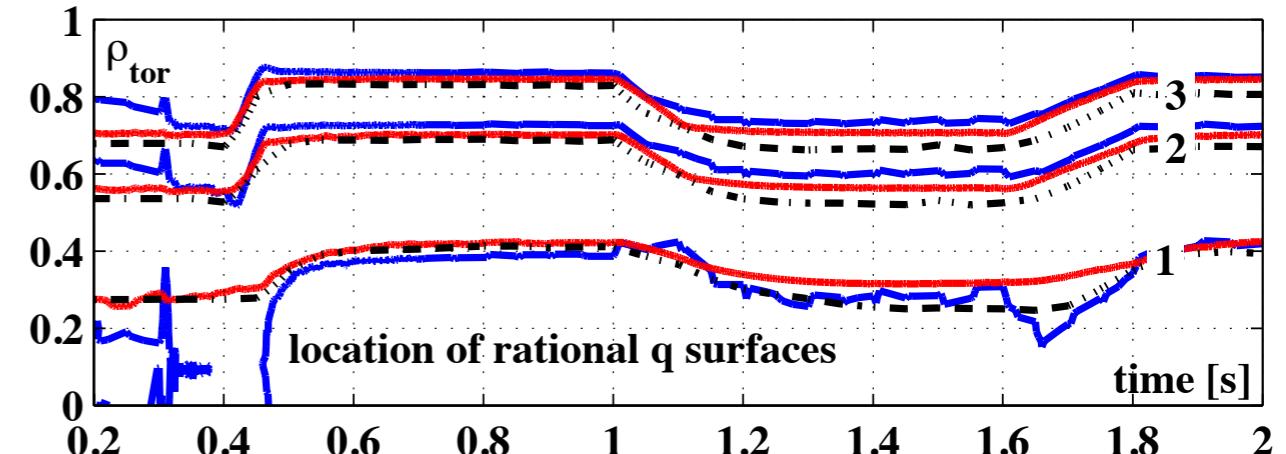
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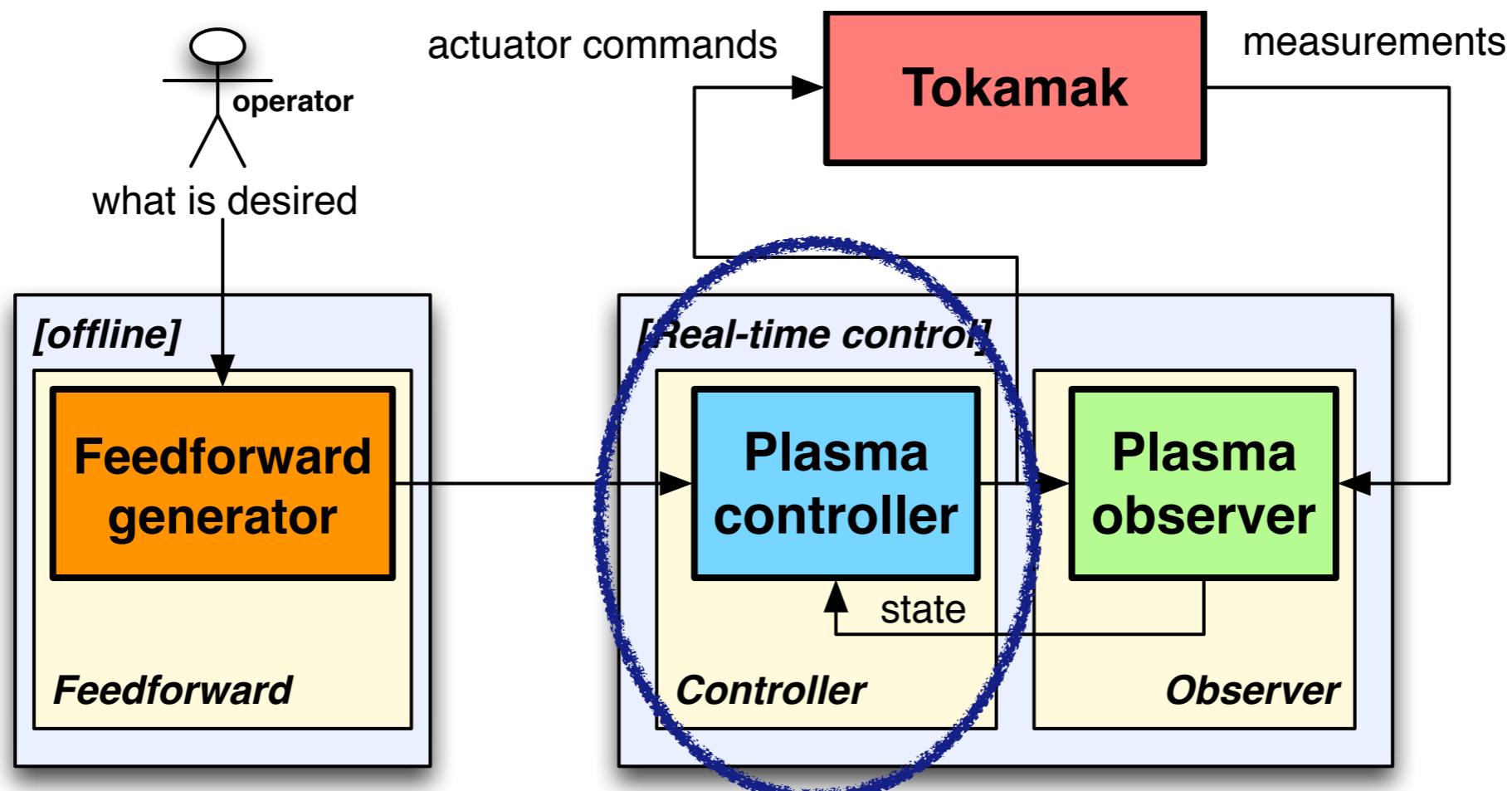
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  - Coupling with GS equilibrium to include magnetics.
  - Classification of model errors, faults, disruption signatures.

# Pilot implementation done on TCV, ASDEX-Upgrade implementation underway

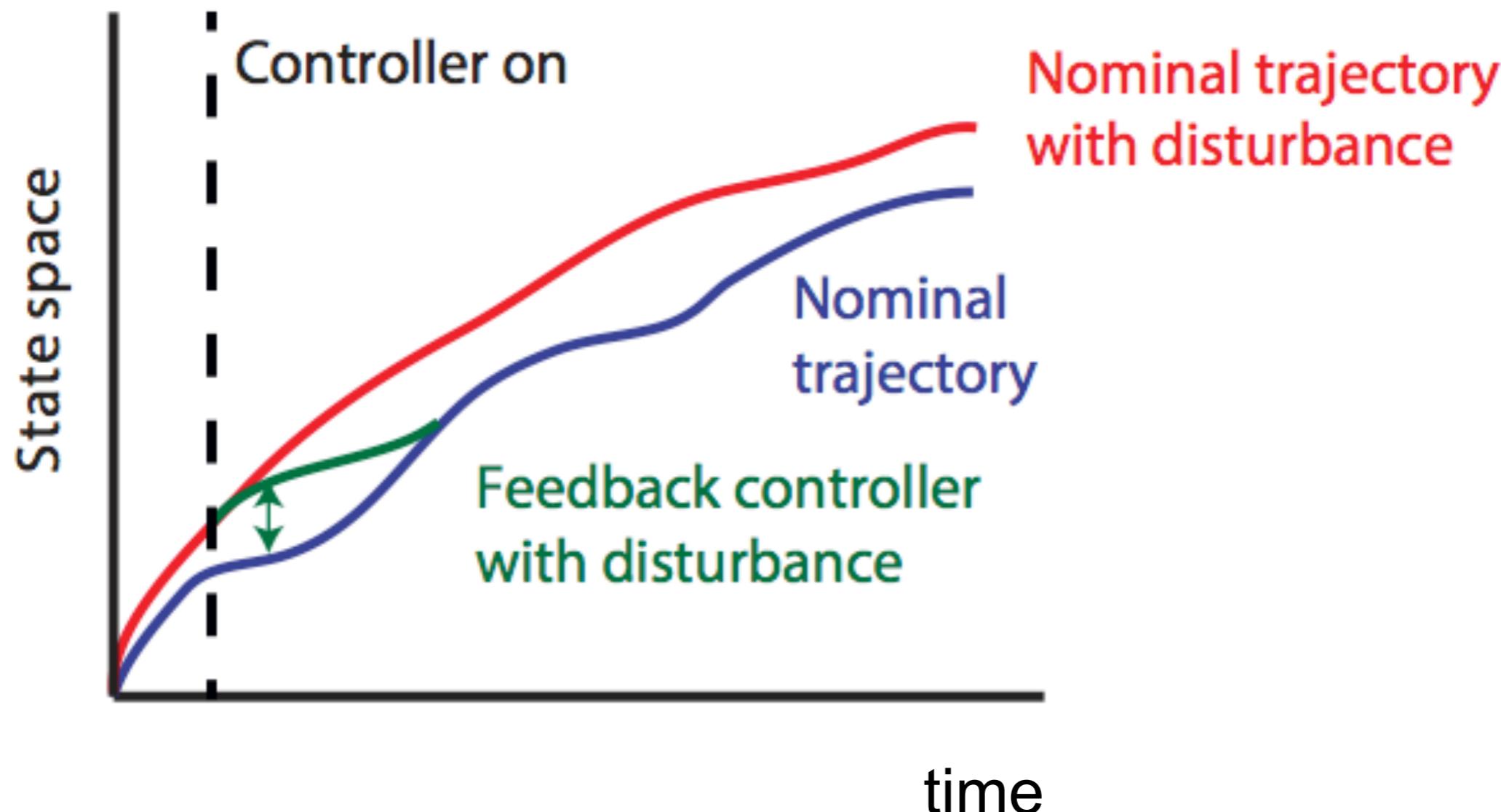
- Pilot RAPTOR implementation solves flux diffusion equation in real-time on TCV real-time control system
  - Kinetic profiles from real-time diagnostics
  - [F. Felici et al, NF2011]



- ASDEX-Upgrade implementation
  - Flux and  $T_e$  evolution, ~3ms per time step
  - Real-time meas. update for  $T_e$  from ECE
  - First results at EPS2013
- ITER simulation proof-of-principle
  - Work to do for this week

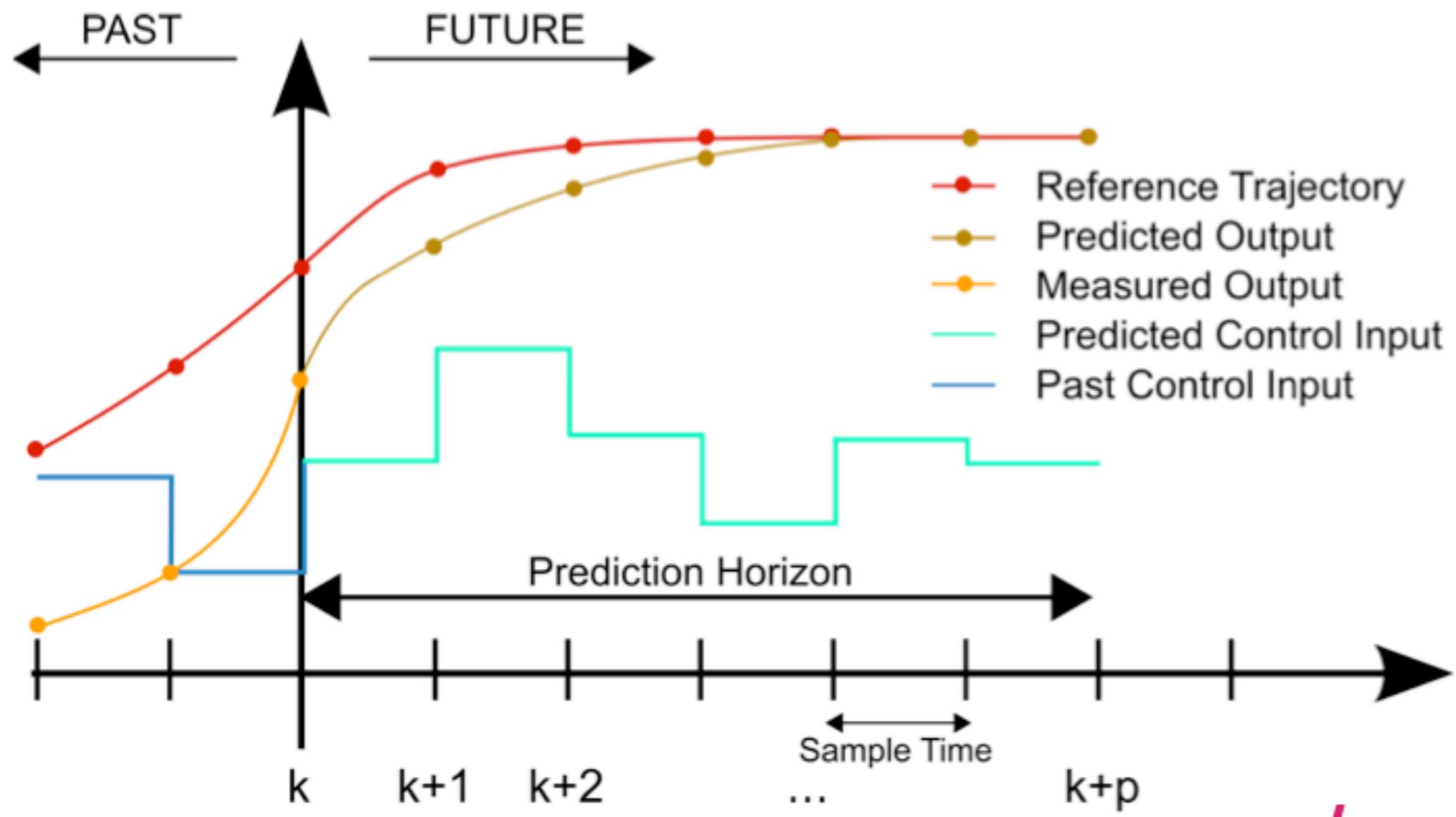


# Feedback control around nominal trajectory, knowing expected variation of profile dynamics



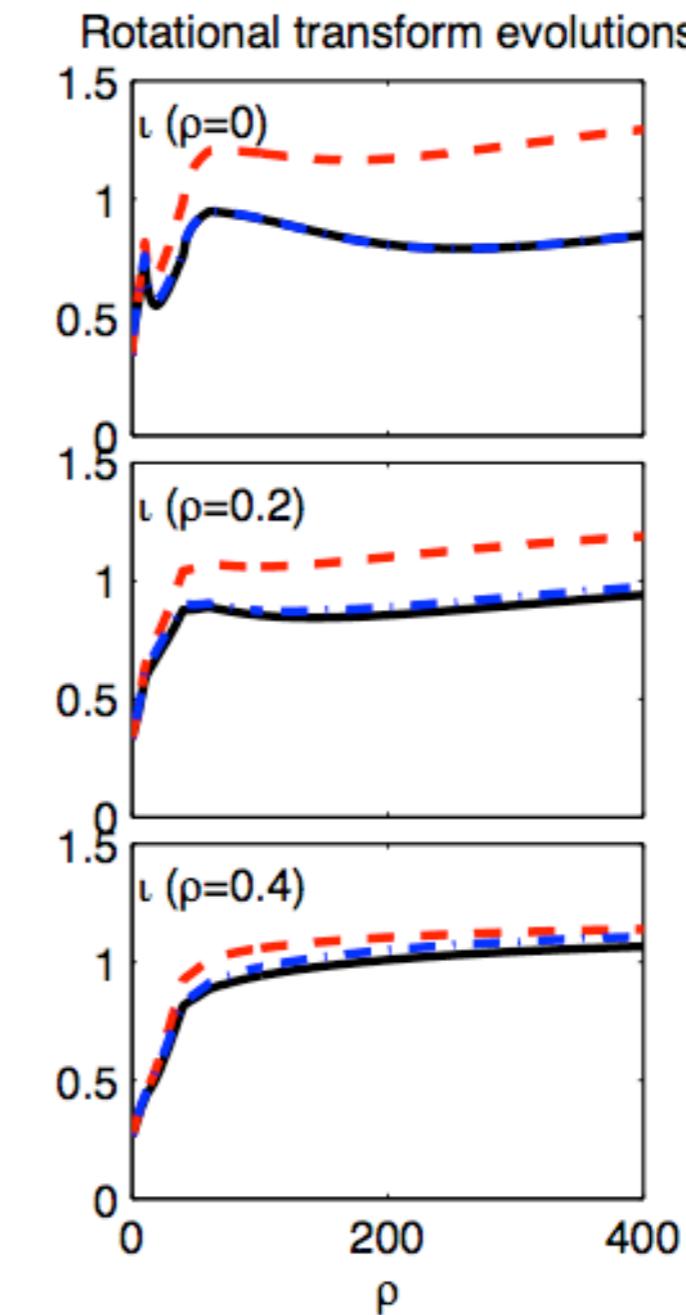
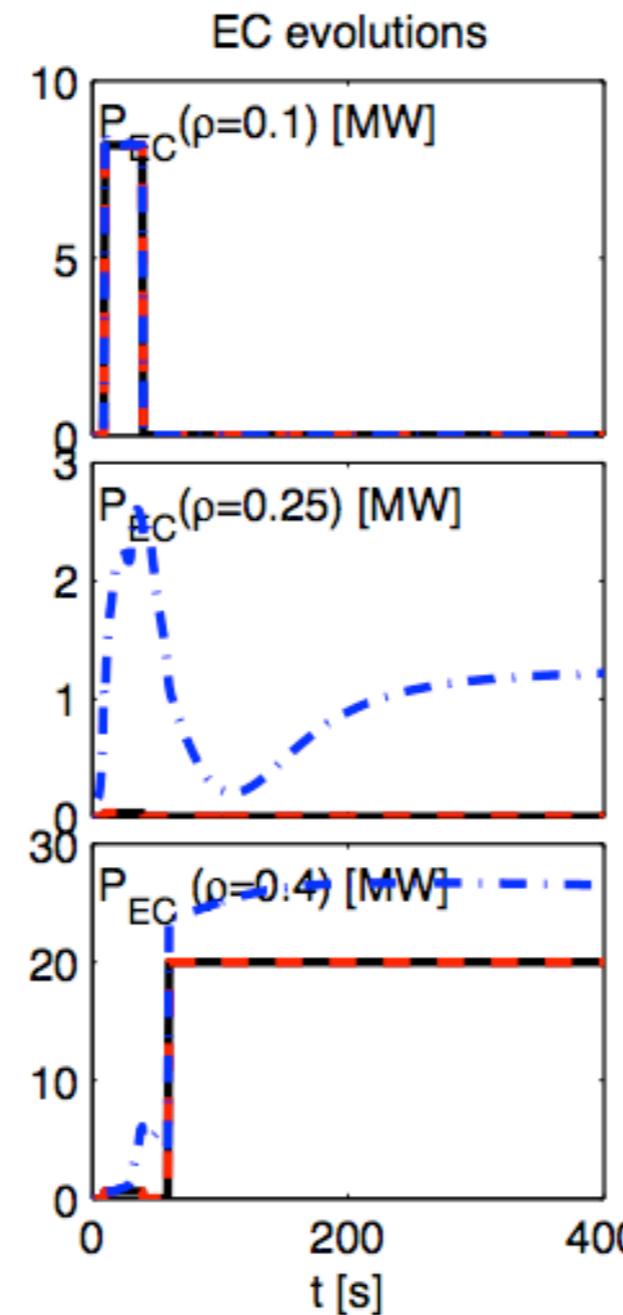
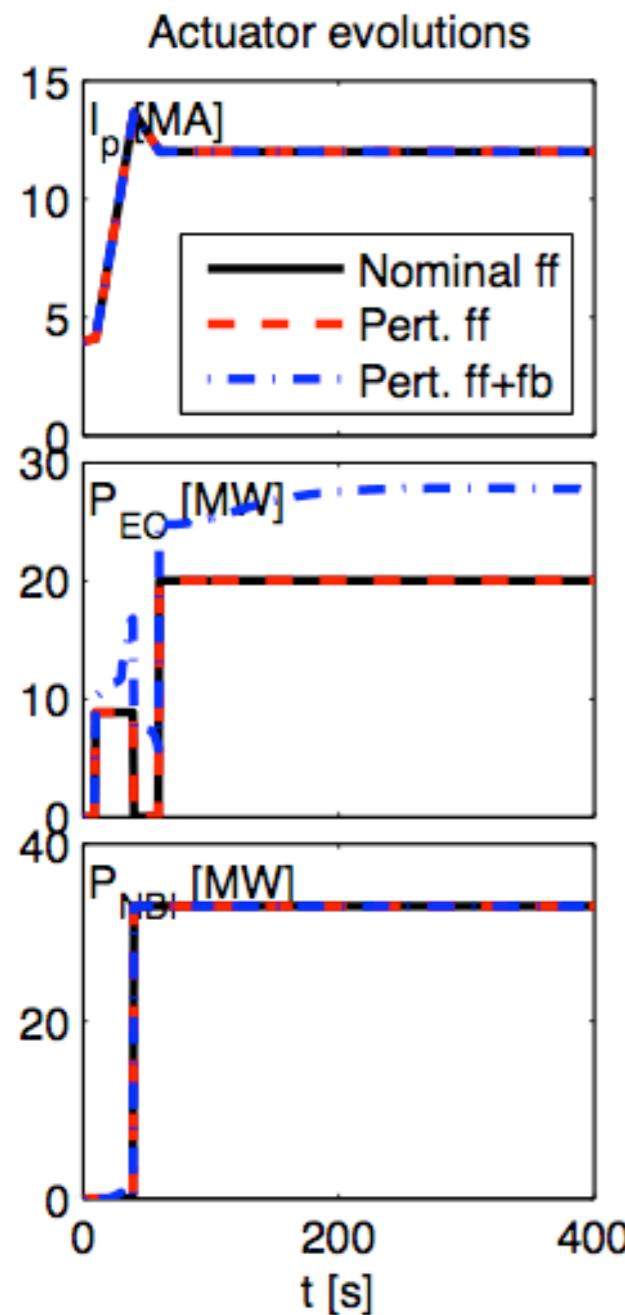
# Model predictive control: determine optimal future actuator trajectory to go back to reference

- Real-time prediction of plasma profile trajectory “for free”
- Naturally include (varying) constraints for state and actuator
- Early warning if constraints can not be met (disruption pred.)



# First results for ITER hybrid scenario show feedback control with model errors, disturbances

- Work by Bert Maljaars (TU/e), to be presented at EPS2013



# Conclusions

- RAPTOR: plasma profile evolution code for real-time control, reconstruction & optimization
  - Key nonlinearities captured in time-evolution
- Model-based optimization of actuator trajectories
  - Numerically compute feedforward trajectories for ramp-up to and ramp-down from flattop.
- Model-based plasma state reconstruction
  - Provides a natural framework to merge diagnostic measurements with model predictions.
- Model-based predictive control
  - Look into the future, control if you can, give warning if you can not
- More details in the literature:
  - [F. Felici, PPCF (2012) 025002]
  - [F. Felici, Nuclear Fusion (2011) 083051]
  - [F. Felici, EPFL Thesis 5203, Lausanne, Switzerland]  
<http://dx.doi.org/10.5075/epfl-thesis-5203>

# Thank you

# Backup slides

# Parameter sensitivity of profile evolution

- Time evolution depends on mode parameters
  - One example: a transport model parameter
  - Another example: a parameter defining the input trajectory

$$\tilde{f}(x_{k+1}, x_k, u_k) = \tilde{f}_k = 0 \quad \forall k$$

- Differentiating with respect to parameter  $p$ , we get the *sensitivity equation*

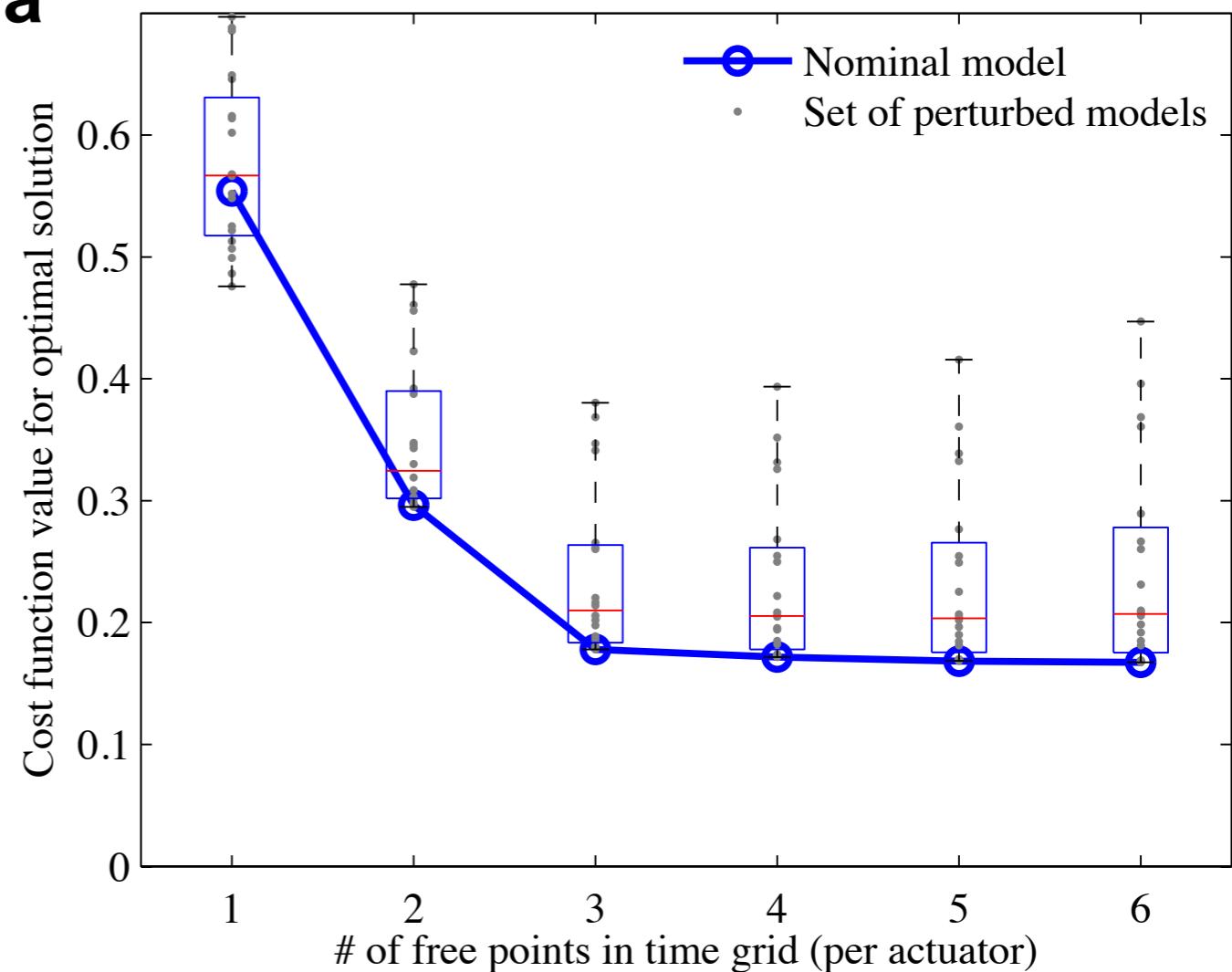
$$0 = \frac{d\tilde{f}_k}{dp} = \frac{\partial \tilde{f}_k}{\partial x_{k+1}} \frac{\partial x_{k+1}}{\partial p} + \frac{\partial \tilde{f}_k}{\partial x_k} \frac{\partial x_k}{\partial p} + \frac{\partial \tilde{f}_k}{\partial u_k} \frac{\partial u_k}{\partial p} + \frac{\partial \tilde{f}_k}{\partial p}$$

- Linear ODE for  $dx_k/dp$ , solve while evolving nonlinear PDE: *Forward sensitivity analysis*
- Jacobians  $df_k/dx_k$ ,  $df_k/dx_{k+1}$  are known from Newton iterations
- Computational cost proportional to  $p$
- $dx_k/dp$  gives the linearization of the state trajectories in the parameter space

$$T_e(\rho, t)|_{p=p_0+\delta p} \approx T_e(\rho, t)_{p_0} + \frac{\partial T_e}{\partial x} \frac{\partial x}{\partial p} \delta p$$

# Multi-grid approach: validate solution against perturbed models to test generalization

- Global nonlinear optimization problem: Risk of local minima
- Multigrid approach
  - Start with 1 free parameter, optimize
  - Increase number of parameters and start from last optimal solution
- Check generalization capabilities of solution by testing against set of perturbed models
  - Little improvement in nominal solution for  $n_f > 4$
  - Degradation in perturbed models for  $n_f > 3$



# Different constraints are active at different times during the ramp-up, consequences for control

- Similar scenario, only  $U_{pl,edge} > 0$  constraint

- Cost function gradient

- Move in this direction to decrease cost

- Constraint gradient

- Move in this direction to violate constraint

- Input arc classification

- Input constrained
  - State constrained
  - Unconstrained

- Consequences for feedback control design

