EFDA

EUROPEAN FUSION DEVELOPMENT AGREEMENT

Task Force
INTEGRATED TOKAMAK MODELLING

IMP-5

Full-wave modelling of electromagnetic wave propagation with the code FWTOR

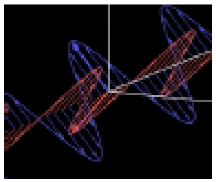
C. Tsironis[1,2], A. Papadopoulos[1], T. Samaras[2], K. Hizanidis[1] and L. Vlahos[2]

[1]School of Electrical and Computer Engineering, National Technical University of Athens, Greece

[2]Department of Physics, Aristotle University of Thessaloniki, Greece

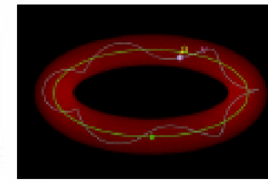
ITM-TF 2010 General Meeting & Training Session, Lisbon-Portugal, 12-17/9/10

Overview of the ECRH/ECCD problem



Wave → Energy transfer → Plasma

Plasma → Instabilities, emission → Wave



EC wave description

Solve the wave equation

1. Asymptotic methods
   a. Ray tracing
   b. Quasi-optics
   c. Beam tracing
2. "Full-wave" methods
   a. (Pseudo) Spectral
   b. Finite differences
   c. Finite elements

System modeling

1. Not self-consistent
   Solve independently
2. Quasi self-consistent
   Solve independently and couple the results
3. Self-consistent
   Solve together (coupled)

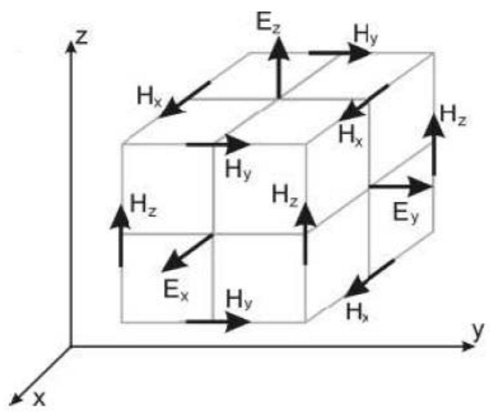
Electron plasma description

Calculate the plasma response

1. Macroscopic description
   a. (Linear) Dielectric tensor
   b. Fluid equations
2. Microscopic description
   a. Fokker-Planck (quasi-linear)
   b. Kinetic-Vlasov (nonlinear)
   c. (Macro) Particle tracing

Finite Difference Time Domain method (FDTD)

- **FDTD provides full-wave solution of Maxwell's equations**
  - Maxwell's curl equations are transformed into *central finite-differences*
  - **In space:** Positioning of the electric and magnetic field on *interlinked contours*, where Faraday and Ampere laws are *valid by identity*
  - **In time:** **E** is computed at a given instant, then **B** is computed at the next instant and so on *(leapfrog scheme)*



- **Despite the advantages and popularity of FDTD, there has been little application to ECRH/ICRH up to now**
  - The required spatial resolution may increase computing needs

IMP-5

FDTD equations in hot anisotropic plasma

- **A separate-field formalism is followed**
  - **$E_t = E_i + E_s$** ***[Total field] = [Incidental field] + [Scattered field]***
    - **Incident field:** **Field that would be present in the absence of the medium**
    - **Scattered field:** **Generated by the medium in response to incident field**
  - **Rationale for approach:** **Incident field can be solved analytically**

- **Maxwell's curl equations (to be discretized and solved…)**

$$\nabla \times \mathbf{E_s} = -\mu_0 \frac{\partial \mathbf{H_s}}{\partial t} \qquad \nabla \times \mathbf{H_s} = \bar{\bar{\sigma}}\mathbf{E_s} + \bar{\bar{\varepsilon}}\frac{\partial \mathbf{E_s}}{\partial t} + \bar{\bar{\sigma}}\mathbf{E_i} + \left(\bar{\bar{\varepsilon}} - \varepsilon_0 \bar{\bar{I}}\right)\frac{\partial \mathbf{E_i}}{\partial t}$$

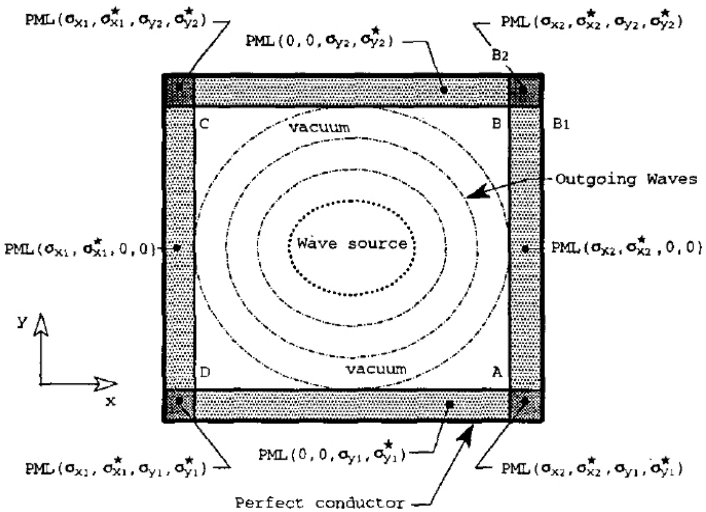
- **Discrete FDTD equations (+ "boundary conditions" = full solution!)**

$$H_{qs}|_{i,j,k}^{n+1/2} = H_{qs}|_{i,j,k}^{n-1/2} - \frac{\Delta t}{\mu_0 \Delta l}\psi_q[\mathbf{E_s}|_{i,j,k}^{n}] \qquad (q = x, y, z)$$

$$E_{qs}|_{i,j,k}^{n+1} = \sum_{l=1}^{3}\sum_{m=1}^{3}\alpha_{ql}|_{i,j,k}\left\{\tau_{lm}|_{i,j,k}E_{ms}|_{i,j,k}^{n} + \psi_q[\mathbf{H_s}|_{i,j,k}^{n+1/2}] - \sigma_{lm}|_{i,j,k}E_{mi}|_{i,j,k}^{n+1/2} - (\varepsilon_{lm}|_{i,j,k} - \varepsilon_0\delta_{lm})\,\frac{\partial E_{mi}}{\partial t}\Big|_{i,j,k}^{n+1/2}\right\}$$

Boundary conditions for outgoing wave propagation

- **For modeling propagation beyond the FDTD space, a special type of boundary conditions is required**
  - Some nearest-neighbor components, needed to evaluate the fields on the boundary, are out of the computational space
- **Types of boundary conditions:**
  - *Outer Radiating (ORBCs):*
    Factorize the wave equation and allow only the solutions for waves outgoing from the problem space
  - ***Absorbing (ABCs):***
    Enclose the computational domain with a lossy material that dampens the outgoing fields



- **For the typical wave & plasma geometries met in ICRH/ECRH, larger accuracy is obtained with ABCs**

Hot-plasma dielectric tensor

$$\tilde{\epsilon} = \tilde{\mathrm{I}} + \frac{\omega_{pe}^2}{\omega} \sum_{l=-\infty}^{\infty} \int \frac{\frac{1}{\gamma p}\frac{df_0}{dp}}{\omega - k_{||}\mathrm{v}_{||} - \frac{l\omega_{ce}}{\gamma}} \begin{bmatrix} \frac{l^2}{\beta_J^2} J_l^2 p_\perp^2 & i\frac{l}{\beta_J} J_l J_l' p_\perp^2 & \frac{l}{\beta_J} J_l^2 p_{||} p_\perp \\ -i\frac{l}{\beta_J} J_l J_l' p_\perp^2 & J_l'^2 p_\perp^2 & -i J_l J_l' p_{||} p_\perp \\ \frac{l}{\beta_J} J_l^2 p_{||} p_\perp & i J_l J_l' p_{||} p_\perp & J_l^2 p_{||}^2 \end{bmatrix} d^3p$$

- **For relativistic plasma, analytic expressions cannot be derived without resorting to approximations**
  - ***Weakly-relativistic model:*** $\gamma \approx 1 + p^2/m^2c^2$, f_0=Juettner distribution

$$\epsilon_{xx} = 1 - \beta_T \frac{\omega_{pe}^2}{\omega^2} \sum_{l=-\infty}^{\infty} \frac{l^2}{\beta_\Gamma} \Gamma_{|l|} \mathcal{S}_{|l|+3/2} \qquad \epsilon_{zz} = 1 - \beta_T \frac{\omega_{pe}^2}{\omega^2} \sum_{l=-\infty}^{\infty} \Gamma_{|l|} \left[\beta_T N_{||}^2 \left(\mathcal{S}_{|l|+7/2} - 2\mathcal{S}_{|l|+5/2} + \mathcal{S}_{|l|+3/2} \right) + \mathcal{S}_{|l|+5/2} \right]$$

$$\epsilon_{xy} = -\epsilon_{yx} = -i\beta_T \frac{\omega_{pe}^2}{\omega^2} \sum_{l=-\infty}^{\infty} l \Gamma'_{|l|} \mathcal{S}_{|l|+3/2} \qquad \epsilon_{xz} = \epsilon_{zx} = \beta_T \frac{\omega_{pe}^2}{\omega \omega_{ce}} N_{||} N_\perp \sum_{l=-\infty}^{\infty} \frac{l}{\beta_\Gamma} \Gamma_{|l|} \left(\mathcal{S}_{|l|+3/2} - \mathcal{S}_{|l|+5/2} \right)$$

$$\epsilon_{yz} = -\epsilon_{zy} = i\beta_T \frac{\omega_{pe}^2}{\omega \omega_{ce}} N_{||} N_\perp \sum_{l=-\infty}^{\infty} \Gamma'_{|l|} \mathcal{S}_{|l|+5/2} \qquad \epsilon_{yy} = 1 - \beta_T \frac{\omega_{pe}^2}{\omega^2} \sum_{l=-\infty}^{\infty} \left(\frac{l^2}{\beta_\Gamma} \Gamma_{|l|} \mathcal{S}_{|l|+3/2} + 2\beta_\Gamma \Gamma'_{|l|} \mathcal{S}_{|l|+5/2} \right)$$

  - ***Question:*** Is it consistent to use the frequency domain tensor in FDTD?
    - The answer is: ***YES***

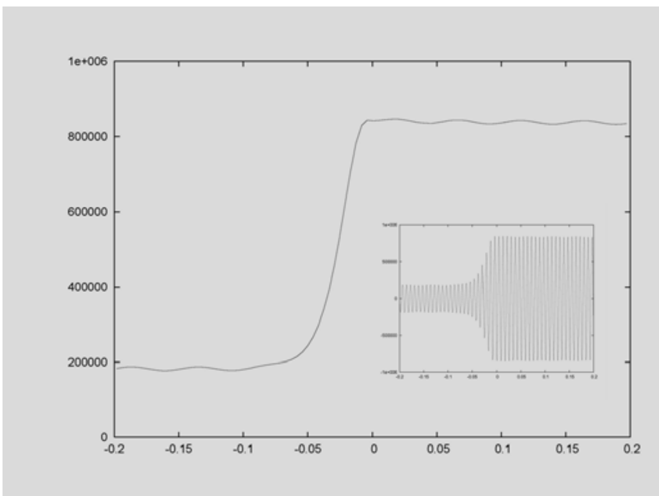
$$\tilde{\epsilon}(\mathbf{r}, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{\epsilon}(\mathbf{r}, \omega_0) e^{-i\omega' t} d\omega' = \tilde{\epsilon}(\mathbf{r}, \omega_0)\delta(t)$$

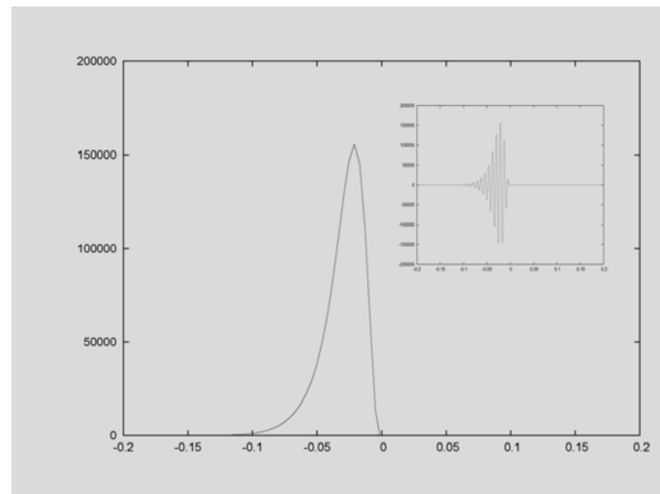
IMP-5

Example:
1D hot-plasma propagation

- **Perpendicular propagation of the X2 mode in AUG plasma**
  - Wave power: $P_0 = 1\mathrm{MW}$
  - Plasma parameters relevant to ASDEX Upgrade

- **Profile of the electric field amplitude and current density**





- **Benchmark with asymptotic solution (beam tracing):** ***Successful!***

Work related to IMP-5 in 2010

- **Finished the development of code FWTOR for the simulation of the propagation and absorption of EC/IC waves in tokamak plasma**
  - 2D serial full-wave code, based on the FDTD method, which follows the propagation of arbitrary wave beams in hot, linear, weakly-relativistic plasma with generic (analytic or experimental) magnetic equilibrium
  - *Benchmarks are performed mainly for testing the correct functionality of the absorbing boundary layer and the physics model of the plasma response under the changes made upon upgrading from the 1-D case*
- **Benchmark phase is almost finished – What follows?**
  - **Installation to ITM GATEWAY (standardize compilation and running)**
    - Tested installation of 1D code: *Successful!*
    - Documentation (Physics & numerical model, ITM specifics)
  - **Initiate code communication with input/output CPOs**

Immediate and near-future plans (2011-12)

- **Complete communication through CPOs and make Kepler actor**
- **Parallelize FWTOR**
- **Combine FDTD with absorption modules based on QL/NL theory**
  - Also implement the fully-relativistic hot plasma dielectric tensor
- **Upgrade FWTOR to study IC/LH waves (also in 3D)**
  - 3D FDTD may be applicable, since IC/LH wavelength is smaller compared to EC
  - Prospect to study time-domain effects (like parametric instabilities)