EFDA
EUROPEAN FUSION DEVELOPMENT AGREEMENT

Task Force
INTEGRATED TOKAMAK MODELLING

Validation and verification of the European Transport Solver

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INTRODUCTION

The European Transport Solver is the modular package developed within the Integrated Tokamak Modelling (ITM) Task Force to perform 1-D simulations of the core plasma. It adopts the modular approach, when external physics modules provide the ETS with equilibrium, transport, sources and non-linear MHD events through the standardised interfaces linked with the ITM agreed data structure. It also adopts several numerical schemas, which can be switched depending on the problem needs to be solved.

At the moment the ETS developing team is concentrated on verification and validation (V&V) of the package. There are several contemporary efforts on ETS V&V activity. First - testing of numerical solvers on analytical examples, using method of manufactured solutions, when exact solutions are compared to analytical ones for a simplified physics model. Second – testing the numerical precision and the conservation properties of ETS solvers , following a systematic V&V roadmap (e.g., continuous / discontinuous transport coefficients, different D/V ratio). Third – benchmarking of ETS against existing transport codes, such as ASTRA, JETTO, CRONOS and TRANSP, when all codes are configured in the same way, share the input and use the same or similar physics modules.

EQUATIONS

PHYSICS EQUATIONS:

all ion equations are solved for components (1:NION)

nonlinearities are treated by iterrations

CURRENT:

$$\sigma_{//}\left(\frac{\partial}{\partial t}-\frac{\rho\dot{B}_0}{2B_0}\cdot\frac{\partial}{\partial\rho}\right)\Psi=\frac{F^2}{\mu_0 B_0\rho}\frac{\partial}{\partial\rho}\left[\frac{V'}{4\pi^2}\left\langle\left|\frac{\nabla\rho}{R}\right|^2\right\rangle\frac{1}{F}\frac{\partial\Psi}{\partial\rho}\right]-\frac{V'}{2\pi\rho}\left(j_{ni,\exp}+j_{ni,imp}\cdot\Psi\right)$$

DENSITIES:

$$\left(\frac{\partial}{\partial t}-\frac{\dot{B}_0}{2B_0}\cdot\frac{\partial}{\partial\rho}\rho\right)(V'n_i)+\frac{\partial}{\partial\rho}\Gamma_i=V'\left(S_{i,\exp}-S_{i,imp}\cdot n_i\right)$$

$$n_e=\sum_{ion}\sum_{Zion}Z_{ion}\cdot n_{Z,ion}+\sum_{imp}\sum_{Zimp}Z_{imp}\cdot n_{Z,imp}$$

TEMPERATURES:

$$\frac{3}{2}\left(\frac{\partial}{\partial t}-\frac{B_0}{2B_0}\cdot\frac{\partial}{\partial\rho}\rho\right)\left(n_iT_iV'^{\frac{5}{3}}\right)+V'^{\frac{2}{3}}\frac{\partial}{\partial\rho}\left(q_i+c_1T_i\Gamma_i\right)=V'^{\frac{5}{3}}\left[Q_{i,\exp}-Q_{i,imp}\cdot T_i+Q_{ei}+Q_{zi}+c_2Q_{\Gamma i}\right]$$

$$\frac{3}{2}\left(\frac{\partial}{\partial t}-\frac{\dot{B}_0}{2B_0}\cdot\frac{\partial}{\partial\rho}\rho\right)\left(n_eT_eV'^{\frac{5}{3}}\right)+V'^{\frac{2}{3}}\frac{\partial}{\partial\rho}\left(q_e+\lambda T_e\Gamma_e\right)=V'^{\frac{5}{3}}\left[Q_{e,\exp}-Q_{e,imp}\cdot T_e+Q_{ie}-c_2Q_{\Gamma e}\right]$$

ROTATION:

$$\left(\frac{\partial}{\partial t}-\frac{\dot{B}_0}{2B_0}\cdot\frac{\partial}{\partial\rho}\rho\right)\left(V'\langle R\rangle m_in_iu_{i,\varphi}\right)+\frac{\partial}{\partial\rho}\Phi_i=V'\left(U_{i,\varphi,\exp}-U_{i,\varphi,imp}\langle R\rangle m_iu_{i,\varphi}+U_{zi,\varphi}\right)+U_{i,\varphi,gyro}$$

FLUXES:

$$\Gamma_i=V'\left\langle|\nabla\rho|^2\right\rangle\left(-D_i\frac{\partial n_i}{\partial\rho}+n_iV_i^{pinch}\right)$$

$$\Gamma_e=\sum_{ion}\sum_{Zion}Z_{ion}\cdot\Gamma_{Zion,ion}+\sum_{imp}\sum_{Zimp}Z_{imp}\cdot\Gamma_{Zimp,imp}$$

$$q_i=V'\left\langle|\nabla\rho|^2\right\rangle\left[n_i\left(-\chi_i\frac{\partial T_i}{\partial\rho}+T_iV_{Ti}^{pinch}\right)\right]$$

$$q_e=V'\left\langle|\nabla\rho|^2\right\rangle\left[n_e\left(-\chi_e\frac{\partial T_e}{\partial\rho}+T_eV_{Te}^{pinch}\right)\right]$$

$$\Phi_i=V'\left\langle|\nabla\rho|^2\right\rangle m_in_i\langle R\rangle\cdot\left(-\chi_{u\varphi,i}\frac{\partial u_{i,\varphi}}{\partial\rho}+u_{i,\varphi}V_{u\varphi,i}^{pinch}\right)+m_i\langle R\rangle u_{i,\varphi}\Gamma_i$$

All transport coefficients and sources are computed by external modules in standardized form, and treated as instances of relevant quantity

TRANSPORT COEFFICIENTS:

$$\sigma_{//}=\sum_{i\,model=1}^{n\,model}\sigma_{//,i\,model}\quad or\quad\sigma_{//}=\sum_{isource=1}^{nsource}\sigma_{//,isource}$$

$$D_i=D_{i,an}+D_{i,NC}+D_{i,ext}+\ldots=\sum_{i\,model=1}^{n\,model}D_{i,i\,model}$$

$$V_i^{pinch}=V_{i,an}^{pinch}+V_{i,NC}^{pinch}+V_{i,ext}^{pinch}+\ldots=\sum_{i\,model=1}^{n\,model}V_{i,i\,model}^{pinch}$$

. . .

SOURCES:

$$j_{ni,\exp}=j_{BS,1}+j_{LH,1}+j_{ICRH,1}+j_{NBI,1}+j_{ECRH,1}+\ldots=\sum_{isource=1}^{nsource}j_{isource,1}$$

$$j_{ni,imp}=j_{BS,2}+j_{LH,2}+j_{ICRH,2}+j_{NBI,2}+j_{ECRH,2}+\ldots=\sum_{isource=1}^{nsource}j_{isource,2}$$

$$S_{i,\exp}=S_{i,n,1}+S_{i,NBI,1}+S_{i,ripple,1}+S_{i,ext,1}+\ldots=\sum_{isource=1}^{nsource}S_{i,isource,1}$$

$$S_{i,imp}=S_{i,n,2}+S_{i,NBI,2}+S_{i,ripple,2}+S_{i,ext,2}+\ldots=\sum_{isource=1}^{nsource}S_{i,isource,2}$$

. . .

BOUNDARY CONDITIONS:

the boundary conditions for every equations can be given by specifying the value, the gradient, the scale length or the flux at the outer boundary... or by specifying the generic coefficients in the form:

$$v(\rho_{bnd})\cdot\left.\frac{\partial n(\rho,t)}{\partial\rho}\right|_{bnd}+u(\rho_{bnd})\cdot n(\rho_{bnd},t)=w(\rho_{bnd})$$

GENERIC FORM OF EQUATIONS:

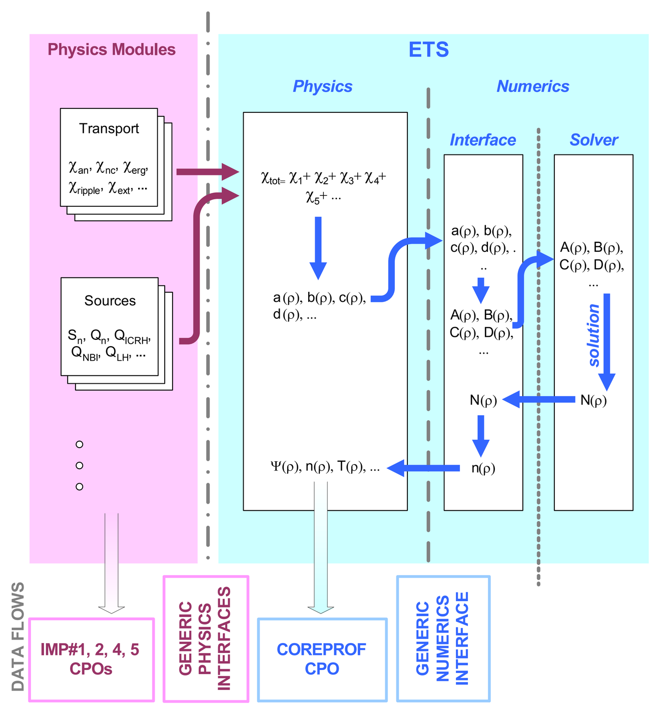
The generic form of equations has been introduced to decouple the physics and numerics parts, this will insure the physics covered by the ETS and simplify the introduction of new numerics

$$\frac{a(\rho)\cdot n(\rho,t)-b(\rho)\cdot n(\rho,t-1)}{h}+\frac{1}{c(\rho)}\frac{\partial}{\partial\rho}\left(-d(\rho)\cdot\frac{\partial n(\rho,t)}{\partial\rho}+e(\rho)\cdot n(\rho,t)\right)=f(\rho)-g(\rho)\cdot n(\rho,t)$$

$$v(\rho_{bnd})\cdot\left.\frac{\partial n(\rho,t)}{\partial\rho}\right|_{bnd}+u(\rho_{bnd})\cdot n(\rho_{bnd},t)=w(\rho_{bnd})$$

CODE STRUCTURE

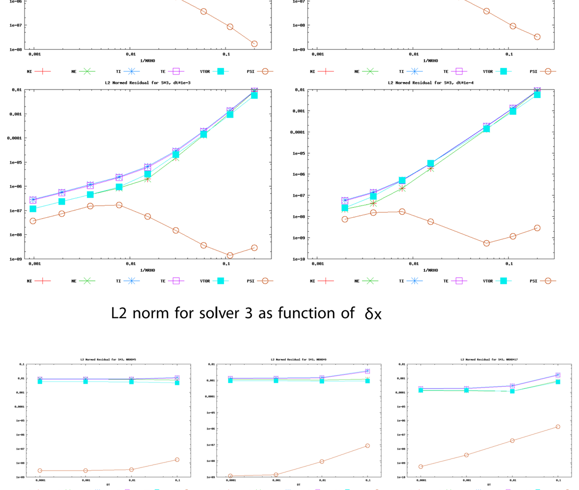
The ETS is designed as a modular package communicating via agreed ITM data base. This allows for easy exchange of modules and benchmarking.



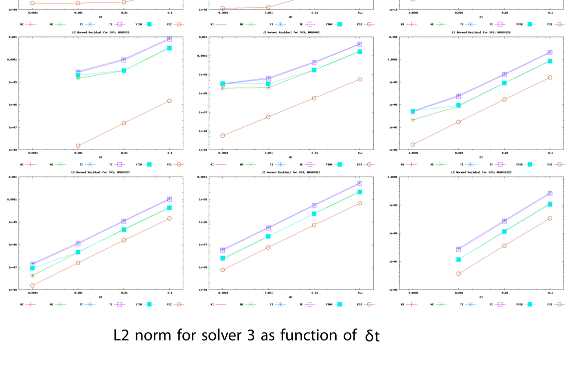
**** All physics modules are receiving the COREPROF CPO as input***

MANUFACTURED SOLUTION TESTS

The goal: First quality check aimed in verifying the coding. The test should prove that the code solves the equations it should be solving with required accuracy.



L2 norm for solver 3 as function of δx



L2 norm for solver 3 as function of δt

Method of manufactured solutions equation:

$$\frac{\partial n}{\partial t}-\frac{1}{\rho}\frac{\partial}{\partial r}\left(\rho D\frac{\partial n}{\partial r}\right)=S$$

Assume some functions for solution and transport coefficient:

$$n=A\rho^2e^{\omega t}\qquad D=B\rho$$

then it is possible to derive the source:

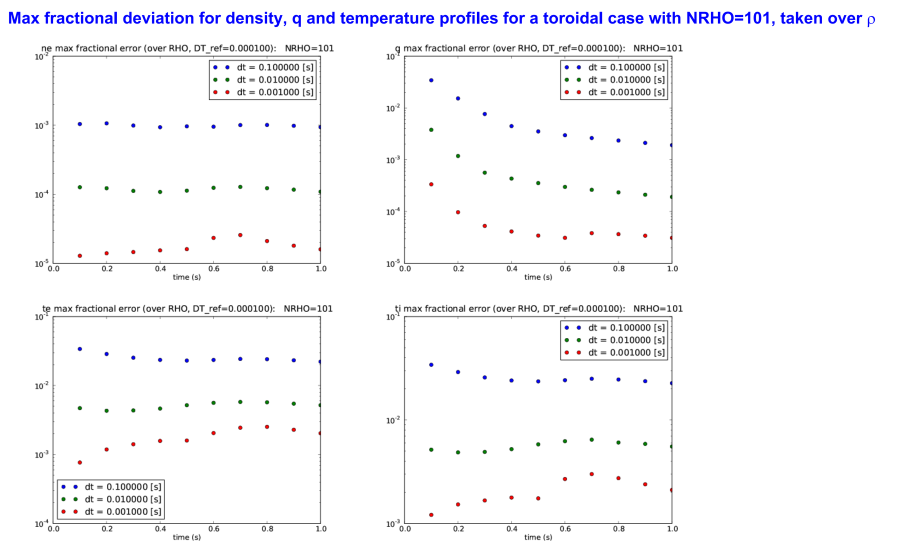
$$S=2A\rho e^{\omega t}\left(\frac{\rho\omega}{2}-3B\right)$$

supplying D and S to the numerical scheme one expects to get back the given function

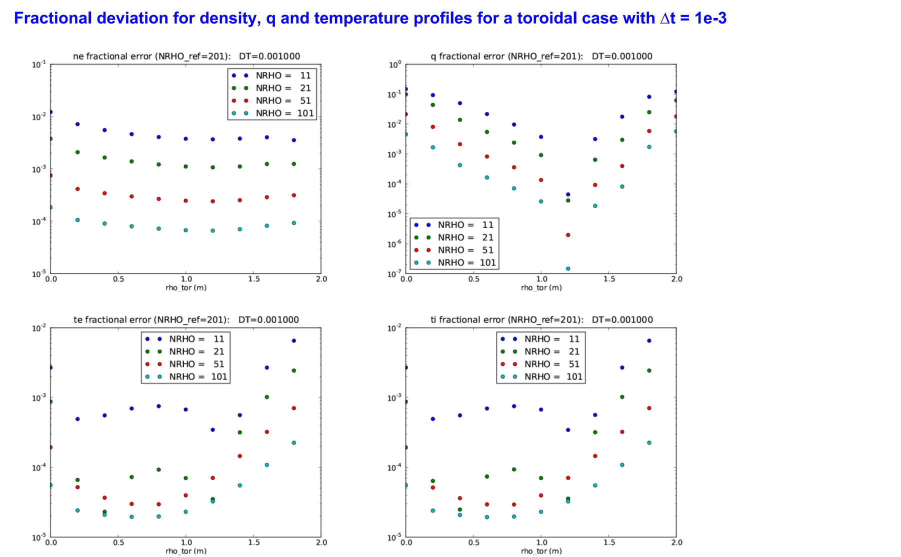
ACCURACY / CONVERGENCE TESTS

The goal: Perform accuracy tests for ETS solver. Study the accuracy of the solver with various time and grid steps $\{\Delta t, \Delta x\}$.

Max fractional deviation for density, q and temperature profiles for a toroidal case with NRHO=101, taken over ρ



Fractional deviation for density, q and temperature profiles for a toroidal case with Δt = 1e-3



DIFFUSION / CONVECTION TESTS

The goal: Study the behaviour of the solver with vrying D/V ratio, find limits on D/V.

Comment: At $D\to 0$ the equation degenerates so that only one the two boundary conditions at $\rho=0$ can be satisfied. Nevertheless, it makes sense to push D in down to zero in order to determine numeric limits and get an idea about residual numerical diffusion of the scheme. For constant v and D the equation $\frac{\partial}{\partial t}\rho n+\frac{\partial}{\partial\rho}\rho\left(vn-D\frac{\partial n}{\partial\rho}\right)=0$ has a steady state (asymptotic at $t\to\infty$) solution which for parabolic initial distribution $n(\rho,t)|_{t=0}=P(n_0,n_1)$ reads

$$n^\infty(\rho)=\frac{n_0+n_1}{4}e^{v\rho/D}\Big/g\left(\frac{va_0}{D}\right),$$

with $g(x)$ being $g(x)=[1+(x-1)e^x]/x^2$ and $g(x)|_{x\to0}\approx\frac{1}{2}+\frac{x}{3}$, $g(x)|_{x\to\infty}\approx\frac{1}{x}e^x$. It is seen that the only parameter that influences the analytic result is va_0/D. Numerically, essential parameter is vh/D (so called grid Peclet number), where h is a size of the space grid cell. It is clear that a reasonable result can be expected if $|vh/D|\ll1$.

Result:

In this example, D has been fixed $D=0.1$ m^2/s, v was varying. For all runs, the quantities Δ_{ne}, Δ_{We} and similar were conserved with the machine accuracy. All equations show similar behaviour therefore we discuss results for the density only. An accuracy of the numerical scheme has been evaluated as $\varepsilon(\rho)=|n_i(\rho,t\to\infty)-n^\infty(\rho)|/n_i(\rho,\infty)$. This quantity shows practically no dependence on ρ and is given in the table below for different values of v.

v	-5	-4	-3	-2	-1	-.3	-.1	.1	.3	1	2
vh/D	-1	-0.8	-0.6	-0.4	-0.2	-0.06	-0.02	0.02	0.06	0.2	0.4
ε, %	4.5	3.2	2.1	1.3	0.83	0.64	0.21	1.1	2.9	9.5	18.5

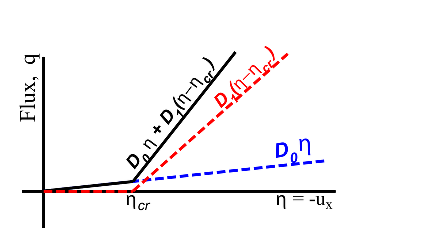
STABILITY (STIFF TRANSPORT)

The goal: Study the stability of numerical scheme for stiff transport models.

Stiff diffusion equation

$$\frac{\partial u}{\partial t}=\frac{\partial}{\partial x}\left(D_0\frac{\partial u}{\partial x}+D_{an}\left(\frac{\partial u}{\partial x}+\eta_{cr}\right)\right)+S$$

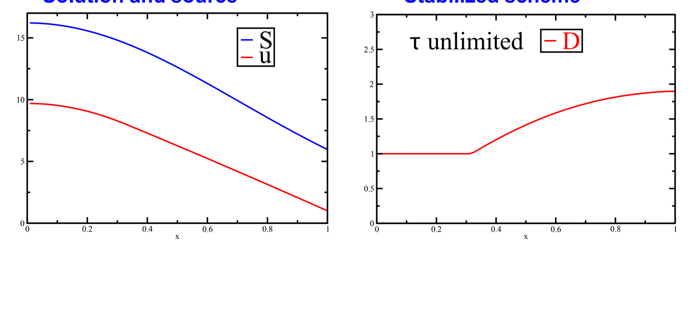
$$D_{an}=\begin{cases}D_1\gg D_0, & \text{if }\left|\frac{\partial u}{\partial x}\right|>\eta_{cr}>0,\\ 0, & \text{if }\left|\frac{\partial u}{\partial x}\right|<\eta_{cr},\end{cases}$$



$$S=5\times10^3e^{-\rho^2/},\qquad D_0=1\text{ m}^2/\text{s},\qquad D_1=5\text{ m}^2/\text{s},\qquad\eta_{cr}=5$$

Preparation tests with ASTRA, ETS needs to be complited

Solution and source / **Stabilized scheme**

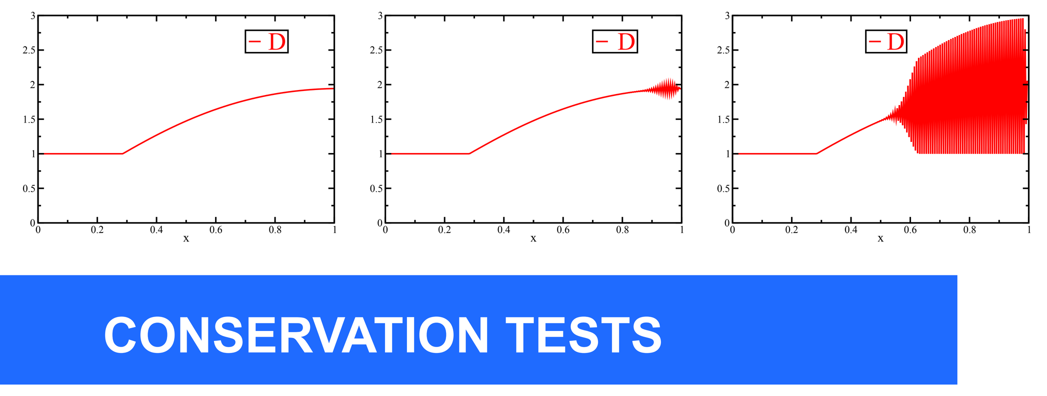


The standard numerical scheme can become unstable with the stiff transport model, even after a minor change of the time resolution.

There are methods to stabilize these numerical oscillations, which need to be integrated in the ETS numerical scheme

Non-stabilized scheme

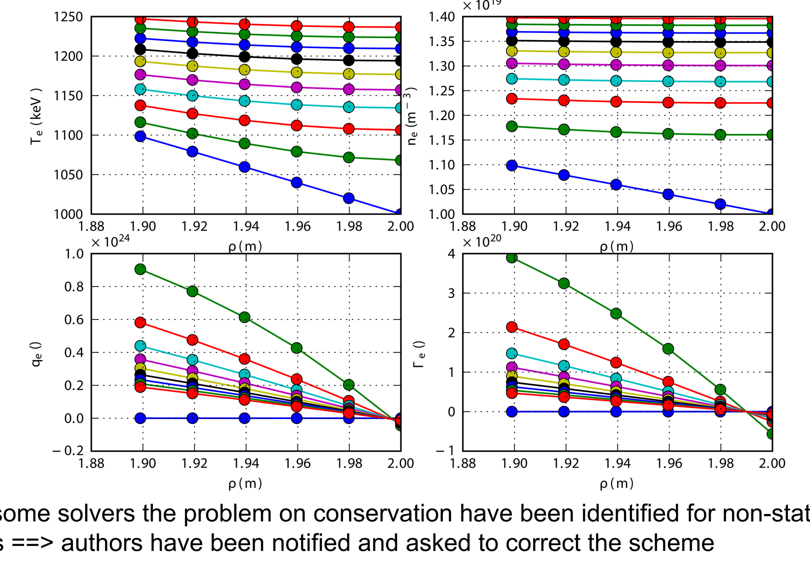
$\tau=1\times10^{-6}$s, (stable) / $\tau=1.1\times10^{-6}$s, (instability start) / $\tau=1.1\times10^{-6}$s, (developed instability)



CONSERVATION TESTS

The goal: 1) To check that various quantities (e.g. total number of particles, total energy) are conserved by the numerical scheme. 2) To check cross-process conservation (e.g., poloidal field disspation and Joule heating)

Case : I.1.5 Solver : 3 N_ρ : 100 N_τ : 101 $\tau=1.0\times10^{-1}$ s



with some solvers the problem on conservation have been identified for non-stationary cases ==> authors have been notified and asked to correct the scheme

BENCHMARKING AGAINST OTHER CODES

The goal: To identify the difference in physics coded in ETS to other existing codes, ASTRA, CRONOS and JETTO, and to create reference data base

Settings for the computations:
Shot: **JET #71827**
Time **52 - 152s**
NRHO **101**

Equations:
Current equation – **predictive** (total current = 2.56 MA)

All **other transport equations**
if interpretative - profiles from t=52 s. are kept through the computations

if predictive - boundary value from t=52 s. is used for the B.C.

Equilibrium (3 moment solvers were used):
D shape model
Major radius = 287 cm
Minor radius = 99 cm
Elongation = 1.65
Triangularity = 0.2
Zaxis = 0
Number of iterations = 50
Geometrical radius = 295 cm
Btor = 2.56 T

Transport:
The resistivity is Spitzer's, all other coefficients are constant

Sources:
prescribed profiles of sources

Present physics capabilities of the ETS:

Equilibrium:
number of fixed boundary equilibrium solvers are integrated in ETS workflow BDSEQ, EMEQ, HELENA

Transport:
ETAIGB, NEOWES, NCLASS

Sources:
generic Gaussian sources, fluid module for puffed neutrals

Impurity:
module for impurity density

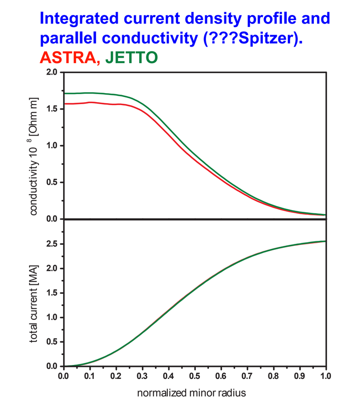
Ifrastructure developments supporting the ETS:

Data base:
object oriented data base for the ETS is compleeted

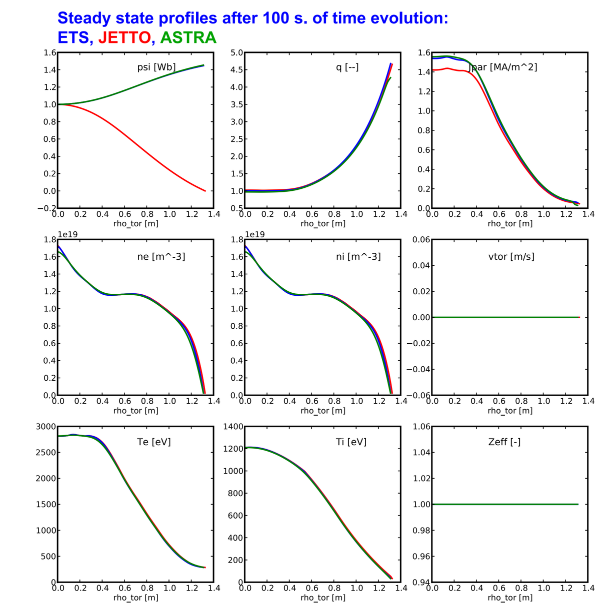
Communicator to machine data bases:
The exp2ITM tool is ready and allows for data translation between JET and ITM

Current equation

Integrated current density profile and parallel conductivity (???Spitzer). ASTRA, JETTO

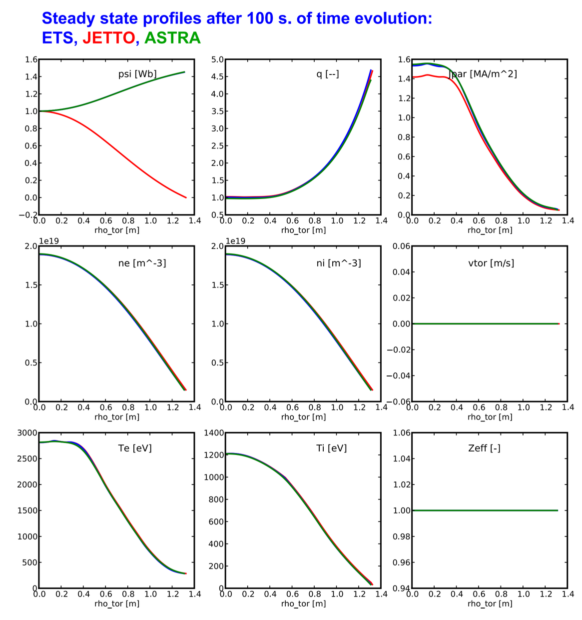


Steady state profiles after 100 s. of time evolution: ETS, JETTO, ASTRA

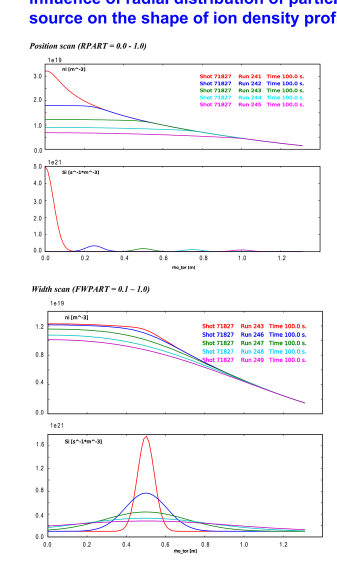


Current + density equations

Steady state profiles after 100 s. of time evolution: ETS, JETTO, ASTRA



Test of generic source: influence of radial distribution of particle source on the shape of ion density profile



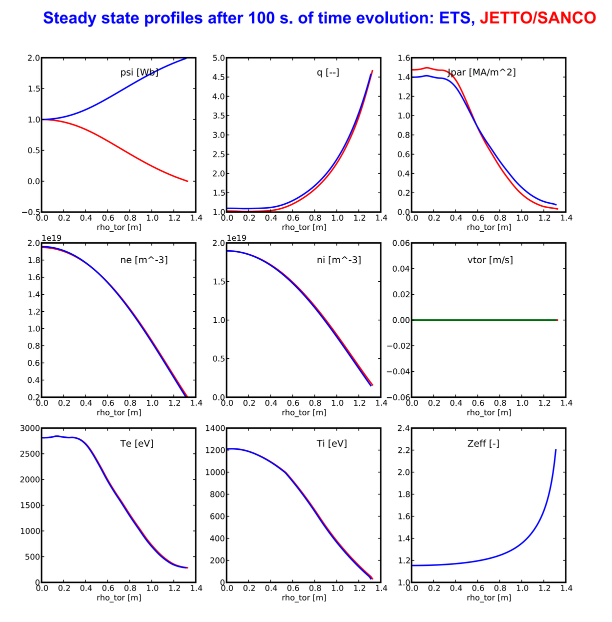
Current + density + carbon density equations

Impurity:

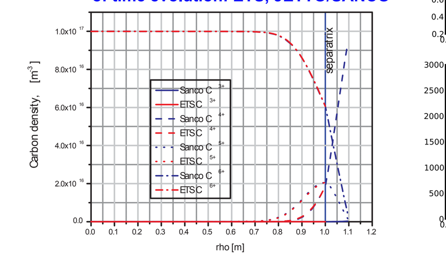
H, D, He, Li, Be, B, C, N, O, Ne, Si, Ar, Mo, Fe, W

boundary conditions are given by total density at the separatrix, assuming coronal equilibrium

Steady state profiles after 100 s. of time evolution: ETS, JETTO/SANCO



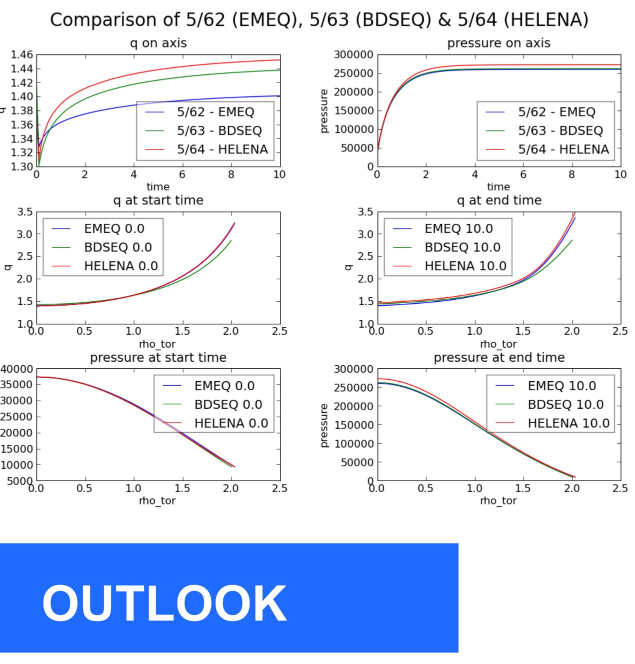
Carbon profiles after 100 s. of time evolution: ETS, JETTO/SANCO



VERIFICATION OF PHYSICS MODULES

The goal: to identify differencies and aplicability ranges of similar physics modules attached to the ETS

Comparison of 5/62 (EMEQ), 5/63 (BDSEQ) & 5/64 (HELENA)



OUTLOOK

Detailed validation and verification strategy has been developed and partially completed. Tests on manufactured solutions have been completed. Numerical tests on D/V, stability and conservation have been started. Benchmarking to ASTRA, JETTO and SANCO codes have been performed for simple cases. The reasonable agreement have been found between different codes, unless some differences in parallel resistivity.
The benchmarking of ETS to other codes will be continued with increasing complexity of physics (transport coefficients and sources from more sophisticated modules).
- current rump up / rump down
- benchmarking of anomalous transport implementation in different codes
- benchmarking of sources implementation in different codes
- predictive modelling of JET discharges