ETS transport equations and list of variables

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Contents

1 Current diffusion equation

outcome:

$\psi(\rho)$ — flux function,
$j_{\parallel}(\rho)$ — parallel current density,
$j_{\mathrm{tor}}(\rho)$ — toroidal current density,
$Q_{\mathrm{OH}}(\rho)$ — ohmic heating power,
$q(\rho)$ — safety factor,
$E_{\parallel}(\rho)$ — parallel electric field

$$\sigma_{\parallel}\left(\frac{\partial}{\partial t}-\frac{\rho\dot{B}_0}{2B_0}\cdot\frac{\partial}{\partial\rho}\right)\Psi=\frac{F^2}{\mu_0 B_0\rho}\frac{\partial}{\partial\rho}\left[\frac{V'}{4\pi^2}\left\langle\left|\frac{\nabla\rho}{R}\right|^2\right\rangle\frac{1}{F}\frac{\partial\Psi}{\partial\rho}\right]-\frac{V'}{2\pi\rho}\left(j_{\mathrm{ni,exp}}+j_{\mathrm{ni,imp}}\cdot\Psi\right) \tag{1.1}$$

where:

μ_0 is permeability of free space ;

B_0 is the magnetic field on geometrical axis, R_0 is the major radius

$F = R\ B_\varphi$ is a diamagnetic function (*comes from equilibrium solver, IMP1*)

metric coefficients such as V' and $\left\langle\left|\frac{\nabla\rho}{R}\right|^2\right\rangle$ should be provided by the equilibrium solver

non inductive current includes contributions **(1:NSOURCE)** in generic form from external sources (*computed by other modules, like ICRH, ECRH, NBI, neutrals and etc.*), in the form of explicit and implicit terms (*index* 1 *is used for explicit parts and index* 2 *for implicit*):

$$j_{\mathrm{ni,exp}}=j_{\mathrm{BS},1}+j_{\mathrm{LH},1}+j_{\mathrm{ICRH},1}+j_{\mathrm{NBI},1}+j_{\mathrm{ECRH},1}+...=\sum_{\mathrm{isource}=1}^{\mathrm{nsource}}j_{\mathrm{isource},1} \tag{1.2}$$

$$j_{\mathrm{ni,imp}}=j_{\mathrm{BS},2}+j_{\mathrm{LH},2}+j_{\mathrm{ICRH},2}+j_{\mathrm{NBI},2}+j_{\mathrm{ECRH},2}+...=\sum_{\mathrm{isource}=1}^{\mathrm{nsource}}j_{\mathrm{isource},2} \tag{1.3}$$

(*individual contributions by various sources should be provided by IMP#5 modules, accept for neutrals and pellets, being the IMP#3 responsibility*)

Poloidal components of the magnetic field and flux function:

$$B_{\mathrm{pol}}=\frac{|\nabla\rho|}{2\pi\ R_0}\frac{\partial\psi}{\partial\rho} \tag{1.4}$$

Parallel electrical conductivity, computed by COLLISIONS module, is used (unless neoclassical value is provided):

$$\sigma_{\parallel}=1.96\frac{e^2 n_e\tau_e}{m_e} \tag{1.5}$$

with

$$\tau_e=\frac{3}{4}\sqrt{\frac{m_e}{2\pi}}\frac{T_e^{\frac{3}{2}}}{n_e e^4\lambda} \tag{1.6}$$

Safety factor:

$$q=\frac{2\pi B_0\rho}{(\partial\Psi/\partial\rho)} \tag{1.7}$$

Total current:

(toroidal)

$$j_{\rm tor} = \frac{2\pi R_0}{\mu_0 V'} \cdot \frac{\partial}{\partial\rho}\left(H\frac{\partial\psi}{\partial\rho}\right) \tag{1.8}$$

$$H = \frac{V'}{4\pi^2} \cdot \left\langle \left(\frac{\nabla\rho}{R}\right)^2 \right\rangle \tag{1.9}$$

(parallel)

$$j_\parallel = \frac{2\pi}{\mu_0 R_0 V'} \cdot \left(\frac{F}{B_0}\right)^2 \frac{\partial}{\partial\rho}\left(\frac{R_0 B_0}{F} H \frac{\partial\psi}{\partial\rho}\right) \tag{1.10}$$

Ohmic heating:

$$Q_{\rm OH} = \sigma_\parallel E_{\parallel^2} \tag{1.11}$$

$$E_\parallel = \frac{1}{\sigma_\parallel}\left(j_\parallel - j_{\rm ni,exp} - j_{\rm ni,imp}\Psi\right) \tag{1.12}$$

1.1 Boundary conditions

(three different options to specify boundary conditions should be available with the transport solver)

on axis $\rho = 0$:

$$\left.\frac{\partial\psi}{\partial\rho}\right|_{\rho=0} = 0 \tag{1.13}$$

at the edge $\rho = \rho_{\rm bnd}$:

type=1 (value)

$$\psi|_{\rho=\rho_{\rm bnd}} = \psi_{\rm bnd} \tag{1.14}$$

type=2 (total current inside $\rho = \rho_{\rm bnd}$)

$$\left.\frac{\partial\psi}{\partial\rho}\right|_{\rho=\rho_{\rm bnd}} = \frac{4\pi^2\mu_0}{V'\left\langle\left|\frac{\nabla\rho}{R}\right|^2\right\rangle} I_{\rm bnd} \tag{1.15}$$

type=3 (loop voltage at $\rho = \rho_{\rm bnd}$)

$$\left.\frac{\partial\psi}{\partial t}\right|_{\rho=\rho_{\rm bnd}} = U_{\rm loop,bnd} \tag{1.16}$$

type=4 (generic)

$$v_{\rm gen}\left.\frac{\partial\psi}{\partial\rho}\right|_{\rm bnd} + u_{\rm gen}\psi_{\rm bnd} = w_{\rm gen} \tag{1.17}$$

If equation 1.1 is not solved (PSI_BND_TYPE=0), q(ρ) should be specified, other quantities are defined as:

$$\Psi = \int_0^{\rho} \frac{2\pi B_0}{q} \rho \partial\rho \tag{1.18}$$

$$j_{\text{tor}} = \frac{2\pi R_0}{\mu_0 V'} \cdot \frac{\partial}{\partial\rho}\left(H \frac{2\pi B_0 \rho}{q}\right) \tag{1.19}$$

$$j_{\parallel} = \frac{2\pi}{\mu_0 R_0 V'} \cdot \left(\frac{F}{B_0}\right)^2 \frac{\partial}{\partial\rho}\left(\frac{R_0 B_0}{F} H \frac{2\pi B_0 \rho}{q}\right) \tag{1.20}$$

$$Q_{\text{OH}} = \frac{1}{\sigma_{\parallel}}\left(j_{\parallel} - j_{\text{ni,exp}} - j_{\text{ni,imp}} \cdot \Psi\right)^2 \tag{1.21}$$

1.2 Translation of the current equation to the generalized form with numerical coefficients

Time discretization:

$$\frac{\Psi - \Psi^-}{\tau} - \frac{\rho \dot{B}_0}{2B_0} \cdot \frac{\partial}{\partial\rho}\Psi = \frac{F^2}{\sigma_{\parallel}\mu_0 B_0 \rho}\frac{\partial}{\partial\rho}\left[\frac{V'}{4\pi^2}\left\langle\left|\frac{\nabla\rho}{R}\right|^2\right\rangle \frac{1}{F}\frac{\partial\Psi}{\partial\rho}\right] - \frac{V'}{2\pi\rho\sigma_{\parallel}}\left(j_{\text{ni,exp}} + j_{\text{ni,imp}} \cdot \Psi\right) \tag{1.22}$$

$$\begin{aligned} \sigma_{\parallel}\frac{\Psi-\Psi^-}{\tau} + \frac{F^2}{\mu_0 B_0 \rho}\frac{\partial}{\partial\rho}\left[-\frac{V'}{4\pi^2}\left\langle\left|\frac{\nabla\rho}{R}\right|^2\right\rangle \frac{1}{F}\frac{\partial\Psi}{\partial\rho} - \frac{\sigma_{\parallel}\mu_0\rho^2}{F^2}\frac{\dot{B}_0}{2} \cdot \Psi\right] = \\ = -\frac{V'}{2\pi\rho} j_{\text{ni,exp}} - \left(\frac{V'}{2\pi\rho} j_{\text{ni,imp}} + \sigma_{\parallel}\frac{\partial}{\partial\rho}\frac{\dot{B}_0}{2}\frac{\sigma_{\parallel}\mu_0\rho^2}{F^2}\right) \cdot \Psi \end{aligned} \tag{1.23}$$

Generalized form of diffusion equation for the quantity used in the ETS:

$$\frac{a(\rho) \cdot Y(\rho) - b(\rho) \cdot Y^{t-1}(\rho)}{h} + \frac{1}{c(\rho)}\frac{\partial}{\partial\rho}\left(-d(\rho) \cdot \frac{\partial Y(\rho)}{\partial\rho} + e(\rho) \cdot Y(\rho)\right) = f(\rho) - g(\rho) \cdot Y(\rho) \tag{1.24}$$

definitions for numerical coefficients in current equation:

$$a(\rho) = \sigma_{\parallel} \tag{1.25}$$

$$b(\rho) = \sigma_{\parallel} \tag{1.26}$$

$$Y^{t-1}(\rho) = \Psi^- \tag{1.27}$$

$$c(\rho) = \frac{\mu_0 B_0 \rho}{F^2} \tag{1.28}$$

$$d(\rho) = \frac{V'}{4\pi^2}\left\langle\left|\frac{\nabla\rho}{R}\right|^2\right\rangle \frac{1}{F} \tag{1.29}$$

$$e(\rho) = -\frac{\sigma_{\parallel}\mu_0\rho^2}{F^2}\frac{\dot{B}_0}{2} \tag{1.30}$$

$$f(\rho) = -\frac{V'}{2\pi\rho} j_{\text{ni,exp}} \tag{1.31}$$

$$g(\rho) = \frac{V'}{2\pi\rho} j_{\text{ni,imp}} + \frac{\dot{B}_0}{2}\sigma_{\parallel}\frac{\partial}{\partial\rho}\frac{\sigma_{\parallel}\mu_0\rho^2}{F^2} \tag{1.32}$$

$$h = \tau \tag{1.33}$$

1.3 Boundary conditions

(three different options to specify boundary conditions should be available with the transport solver)

general form required by numerical solver:

$v \frac{\partial \psi}{\partial \rho}|_{\text{bnd}} + u\psi_{\text{bnd}} = w$, *where v(1:2), u(1:2), w(1:2) are coefficients required by the solver*

on axis $\rho = 0$:

$$\left.\frac{\partial \psi}{\partial \rho}\right|_{\rho=0} = 0 \quad (1.34)$$
$$v(1) = 1 \quad (1.35)$$
$$u(1) = 0 \quad (1.36)$$
$$w(1) = 0 \quad (1.37)$$

at the edge $\rho = \rho_{\text{bnd}}$:

type=1 (value) PSI_BND_TYPE(2)=1; PSI_BND(2,1)= ψ_{bnd}

$$\psi|_{\rho=\rho_{\text{bnd}}} - \psi_{\text{bnd}} = 0 \quad (1.38)$$
$$v(2) = 0 \quad (1.39)$$
$$u(2) = 1 \quad (1.40)$$
$$w(2) = \psi_{\text{bnd}} \quad (1.41)$$

type=2 (total current inside $\rho = \rho_{\text{bnd}}$**)** PSI_BND_TYPE(2)=2; PSI_BND(2,1)= I_{bnd}

$$\left.\frac{\partial \psi}{\partial \rho}\right|_{\rho=\rho_{\text{bnd}}} = \frac{4\pi^2 \mu_0}{V' \left\langle \left| \frac{\nabla \rho}{R} \right|^2 \right\rangle} I_{\text{bnd}} \quad (1.42)$$
$$v(2) = 1 \quad (1.43)$$
$$u(2) = 0 \quad (1.44)$$
$$w(2) = \frac{4\pi^2 \mu_0}{V' \left\langle \left| \frac{\nabla \rho}{R} \right|^2 \right\rangle} I_{\text{bnd}} \quad (1.45)$$

type=3 (loop voltage at $\rho = \rho_{\text{bnd}}$**)** PSI_BND_TYPE(2)=3; PSI_BND(2,1)=$U_{\text{loop,bnd}}$

$$\psi|_{\rho=\rho_{\text{bnd}}} - \left(\tau U_{\text{loop,bnd}} + \psi^-_{\text{bnd}}\right) = 0 \quad (1.46)$$
$$v(2) = 0 \quad (1.47)$$
$$u(2) = 1 \quad (1.48)$$
$$w(2) = \tau U_{\text{loop,bnd}} + \psi^-_{\text{bnd}} \quad (1.49)$$

type=4 (generic) PSI_BND_TYPE(2)=4; PSI_BND(2,1)=v_{gen}; PSI_BND(2,2)=u_{gen}; PSI_BND(2,3)=w_{gen}

$$v_{\text{gen}} \left.\frac{\partial \Psi}{\partial \rho}\right|_{\text{bnd}} + u_{\text{gen}} \Psi_{\text{bnd}} = w_{\text{gen}} \quad (1.50)$$
$$v(2) = v_{\text{gen}} \quad (1.51)$$
$$u(2) = u_{\text{gen}} \quad (1.52)$$
$$w(2) = w_{\text{gen}} \quad (1.53)$$

1.4 Variables used in ETS (SUBROUTINE CURRENT)

Variable	TYPE%NAME used in ETS data flow	Internal name used in CURRENT routine	Units
τ	EVOLUTION%TAU	TAU	s
$\dot{B}_0$	GEOMETRY%BTPRIME	BTPRIME	T/s
B_0	GEOMETRY%BT	BT	T
R_0	GEOMETRY%R0	R0	m
ρ	GEOMETRY%RHO	RHO	m
V'	GEOMETRY%VPR	VPR	m^2
$\left\langle \left\| \frac{\nabla\rho}{R} \right\|^2 \right\rangle$	GEOMETRY%G3	G3	m^{-2}
F	GEOMETRY%FDIA	FDIA	T·m
H	--	H	
π	--	PI	
μ_0	--	MU0	H/m
Ψ	PROFILES%PSI	PSI	V·s
Ψ^-	EVOLUTION%PSIM	PSIM	V·s
q	PROFILES%QSF	QSF	--
j_{tor}	PROFILES%CURR_TOR	CURR_TOR	A/m^2
$j_{\parallel}$	PROFILES %CURR_PAR	CURR_PAR	A/m^2
$j_{\mathrm{ni,exp}}$	--	CURR_NI_EXP	A/m^2
$j_{\mathrm{ni,imp}}$	--	CURR_NI_IMP	A/(V·s·m^2)
$j_{\mathrm{source,1}}$	SOURCES%CURR_NI_EXP	--	A/m^2
$j_{\mathrm{source,2}}$	SOURCES %CURR_NI_IMP	--	A/(V·s·m^2)
Q_{OH}	SOURCES%QOH	QOH	W/m^3
$\sigma_{\parallel}$	TRANSPORT%SIGMA *or* SOURCES%SIGMA *or* COLLISIONS%SIGMA	SIGMA	(Ohm·m)$^{-1}$
$E_{\parallel}$	PROFILES %E_PAR	E_PAR	V/m
a	SOLVER%A	A	--
b	SOLVER%B	B	--
c	SOLVER%C	C	--
d	SOLVER%D	D	--
e	SOLVER%E	E	--
f	SOLVER%F	F	--
g	SOLVER%G	G	--

h	SOLVER%H	H	--
v(1:2)	SOLVER%V	V	--
u(1:2)	SOLVER%U	U	--
w(1:2)	SOLVER%W	W	--
Y^{t-1}	SOLVER%YM	W	--
solution	SOLVER%Y	Y	--
derivative of solution	SOLVER%DY	DY	--

Functions	**Internal name used in CURRENT routine**
$\frac{\sigma_{\parallel}\mu_0\rho^2}{F^2}$	FUN1
$\frac{\partial}{\partial\rho}\frac{\sigma_{\parallel}\mu_0\rho^2}{F^2}$	DFUN1
$\frac{V'}{4\pi^2}\left\langle\left\|\frac{\nabla\rho}{R}\right\|^2\right\rangle$	FUN2
$\frac{2\pi B_0}{q}$	FUN3
$H\frac{\partial\psi}{\partial\rho}$	FUN4
$\frac{\partial}{\partial\rho}\left(H\frac{\partial\psi}{\partial\rho}\right)$	DFUN4
$\frac{R_0B_0}{F}H\frac{\partial\psi}{\partial\rho}$	FUN5
$\frac{\partial}{\partial\rho}\left(\frac{R_0B_0}{F}H\frac{\partial\psi}{\partial\rho}\right)$	DFUN5

2 Ion density (1:nion)

outcome:

$n_i(\rho, i_{\text{ion}})$ — ion density
$\Gamma_i(\rho, i_{\text{ion}})$ — ion flux
$\gamma_i(\rho, i_{\text{ion}})$ — ion flux contributing to heat transport
$\Gamma_{\text{Si}}(\rho, i_{\text{ion}})$ — integral of sources (flux)

Ion diffusion equation to be solved:

$$\left(\frac{\partial}{\partial t} - \frac{\dot{B}_0}{2B_0} \cdot \frac{\partial}{\partial \rho}\rho\right)(V' n_i) + \frac{\partial}{\partial \rho}\Gamma_i = V' \ \left(S_{i,\text{exp}} - S_{i,\text{imp}} \cdot n_i\right) \tag{2.1}$$

where the total flux defined as:

$$\Gamma_i = V' \left\langle |\nabla\rho|^2 \right\rangle \left(- D_i \frac{\partial n_i}{\partial \rho} + n_i \ V_i^{\text{pinch}} \right) \tag{2.2}$$

where total transport coefficients are defined as a sum of individual contributions **(1:NMODEL)**:

$$D_i = D_{i,\text{an}} + D_{i,\text{NC}} + D_{i,\text{ext}} + ... = \sum_{i\text{model}=1}^{n\text{model}} D_{i,i\text{model}} \tag{2.3}$$

$$V_i^{\text{pinch}} = V_{i,\text{an}}^{\text{pinch}} + V_{i,\text{NC}}^{\text{pinch}} + V_{i,\text{ext}}^{\text{pinch}} + ... = \sum_{i\text{model}=1}^{n\text{model}} V_{i,i\text{model}}^{\text{pinch}} \tag{2.4}$$

(*individual contributions,* $D_{i,i\text{model}}$ and $V_{i,i\text{model}}^{\text{pinch}}$, *to* D_i *and* V_i^{pinch} *by various transport models should be provided by IMP#4 modules*)

and sources include contributions **(1:NSOURCE)** in generic form from NBI, recycling and puffed neutrals, ripple, ergodization, and other possible sources, in the form of explicit and implicit terms (*index* 1 *is used for explicit parts and index* 2 *for implicit*):

$$S_{i,\text{exp}} = S_{i,n,1} + S_{i,\text{NBI},1} + S_{i,\text{ripple},1} + S_{i,\text{ext},1} + ... = \sum_{\text{isource}=1}^{\text{nsource}} S_{i,\text{isource},1} \tag{2.5}$$

$$S_{i,\text{imp}} = S_{i,n,2} + S_{i,\text{NBI},2} + S_{i,\text{ripple},2} + S_{i,\text{ext},2} + ... = \sum_{\text{isource}=1}^{\text{nsource}} S_{i,\text{isource},2} \tag{2.6}$$

(*individual contributions by various sources should be provided by IMP#5 modules, accept for neutrals and pellets, being the IMP#3 responsibility*)

2.1 Boundary conditions

Following options to specify boundary conditions should be available with the transport solver

NOTE! Specified positive values for $\nabla n_{i,\text{bnd}}$ ***and*** L_{ni} ***correspond to "normal" profile with density decreasing towards the edge***

on axis $\rho = 0$:

$$\frac{\partial n_i}{\partial \rho}|_{\rho=0} = 0 \tag{2.7}$$

$$V_i^{\text{pinch}} = 0 \tag{2.8}$$

at the edge $\rho = \rho_{\text{bnd}}$:

type=1 (value)

$$n_i|_{\rho=\rho_{\text{bnd}}} = n_{i,\text{bnd}} \tag{2.9}$$

type=2 (gradient)

$$\frac{\partial n_i}{\partial \rho}|_{\rho=\rho_{\text{bnd}}} = -\nabla n_{i,\text{bnd}} \tag{2.10}$$

type=3 (scale length)

$$\frac{1}{(\partial \ln n_i / \partial \rho)}|_{\rho=\rho_{\text{bnd}}} = -L_{\text{ni}} \tag{2.11}$$

type=4 (flux)

$$V' \left\langle |\nabla\rho|^2 \right\rangle \left(- D_i \frac{\partial n_i}{\partial \rho} + n_i \, V_i^{\text{pinch}} \right) |_{\rho=\rho_{\text{bnd}}} = \Gamma_{i,\text{bnd}} \tag{2.12}$$

type=5 (generic)

$$v_{\text{gen}} \left. \frac{\partial n_i}{\partial \rho} \right|_{\text{bnd}} + u_{\text{gen}} n_{i,\text{bnd}} = w_{\text{gen}} \tag{2.13}$$

If equation 2.1 is not solved (NI_BND_TYPE=0), interpretative density should be specified and flux is calculated from the integral of sources:

$$n_i = n_{i,\text{interp}} \tag{2.14}$$

$$\Gamma_i = \Gamma_{\text{Si}} \tag{2.15}$$

$$\gamma_i = \frac{3}{2} \Gamma_{\text{Si}} \tag{2.16}$$

where τ is the time step, V'^{-} and n_{i^-} are taken from the previous time step, and

$$\Gamma_{\text{Si}} = \frac{\dot{B}_0}{2B_0} \cdot \left(\rho V' n_{i,\text{interp}} \right) + \int_0^{\rho} \left(V' S_{i,\text{exp}} + \frac{V'^{-} n_{i,\text{interp}^-}}{\tau} - n_{i,\text{interp}} V' \cdot \left(\frac{1}{\tau} + S_{i,\text{imp}} \right) \right) \partial\rho \tag{2.17}$$

2.2 Translation of the ion density equation to the generalized form with numerical coefficients

$$\frac{\partial}{\partial t} (V' n_i) - \frac{\dot{B}_0}{2B_0} \cdot \frac{\partial}{\partial \rho} (\rho V' n_i) + \frac{\partial}{\partial \rho} \left[V' \left\langle |\nabla\rho|^2 \right\rangle \left(- D_i \frac{\partial n_i}{\partial \rho} + n_i \, V_i^{\text{pinch}} \right) \right] = V' \, (S_{i,\text{exp}} - S_{i,\text{imp}} \cdot n_i) \tag{2.18}$$

$$\begin{aligned} \frac{\partial}{\partial t} (V' n_i) + \frac{\partial}{\partial \rho} \left[V' \left\langle |\nabla\rho|^2 \right\rangle \left(- D_i \frac{\partial n_i}{\partial \rho} + n_i \, V_i^{\text{pinch}} \right) - \frac{\dot{B}_0}{2B_0} \cdot (\rho V' n_i) \right] = \\ = V' \, S_{i,\text{exp}} - V' n_i \cdot \left[\rho \frac{\partial}{\partial \rho} \left(\frac{\dot{B}_0}{2B_0} \right) + S_{i,\text{imp}} \right] \end{aligned} \tag{2.19}$$

time discretization:

$$\frac{\partial V' n_i}{\partial t} = \frac{V' n_i - V'^{-} n_{i^-}}{\tau} \tag{2.20}$$

where V'^{-} and n_{i^-} are taken at the previous time step

$$\begin{aligned} \frac{V' n_i - V'^{-} n_{i^-}}{\tau} + \frac{\partial}{\partial \rho}\left[-V'\left\langle|\nabla\rho|^2\right\rangle D_i \frac{\partial n_i}{\partial \rho} + \left(V'\left\langle|\nabla\rho|^2\right\rangle V_i^{\text{pinch}} - \frac{\dot{B}_0}{2B_0}\cdot\rho V'\right)\cdot n_i\right] = \\ = V' \, S_{i,\text{exp}} - V'\left[\rho\frac{\partial}{\partial\rho}\left(\frac{\dot{B}_0}{2B_0}\right) + S_{i,\text{imp}}\right]\cdot n_i \end{aligned} \tag{2.21}$$

Generalized form of diffusion equation for the quantity used in the ETS:

$$\frac{a(\rho)\cdot Y(\rho) - b(\rho)\cdot Y^{t-1}(\rho)}{h} + \frac{1}{c(\rho)}\frac{\partial}{\partial\rho}\left(-d(\rho)\cdot\frac{\partial Y(\rho)}{\partial\rho} + e(\rho)\cdot Y(\rho)\right) = f(\rho) - g(\rho)\cdot Y(\rho) \tag{2.22}$$

definitions for numerical coefficients in ion density equation:

$$a(\rho) = V' \tag{2.23}$$

$$b(\rho) = V'^{-} \tag{2.24}$$

$$Y^{t-1}(\rho) = n_i^{-} \tag{2.25}$$

$$c(\rho) = 1 \tag{2.26}$$

$$d(\rho) = V'\left\langle|\nabla\rho|^2\right\rangle D_i \tag{2.27}$$

$$e(\rho) = V'\left\langle|\nabla\rho|^2\right\rangle V_i^{\text{pinch}} - \frac{\dot{B}_0}{2B_0}\cdot\rho V' \tag{2.28}$$

$$f(\rho) = V' S_{i,\text{exp}} \tag{2.29}$$

$$g(\rho) = V' S_{i,\text{imp}} \tag{2.30}$$

$$h = \tau \tag{2.31}$$

2.3 Boundary conditions

generalized form required by numerical solver:

$v\frac{\partial Y}{\partial\rho}|_{\text{bnd}} + \text{uY}_{\text{bnd}} = w$, *where v(1:2), u(1:2), w(1:2) are coefficients required by the solver*

on axis $\rho = 0$:

$$\frac{\partial n_i}{\partial \rho}|_{\rho=0} = 0 \tag{2.32}$$

$$v(1) = 1 \tag{2.33}$$

$$u(1) = 0 \tag{2.34}$$

$$w(1) = 0 \tag{2.35}$$

$$e(1) = -\frac{\dot{B}_0}{2B_0}\cdot\rho V' \tag{2.36}$$

at the edge $\rho = \rho_{\text{bnd}}$:

type=1 (value) NI_BND_TYPE(2)=1; NI_BND(2,1)=$n_{i,\mathrm{bnd}}$

$$n_i|_{\rho=\rho_{\mathrm{bnd}}} = n_{i,\mathrm{bnd}} \tag{2.37}$$
$$u(2) = 0 \tag{2.38}$$
$$v(2) = 1 \tag{2.39}$$
$$w(2) = n_{i,\mathrm{bnd}} \tag{2.40}$$

type=2 (gradient) NI_BND_TYPE(2)=2; NI_BND(2,1)=$\nabla n_{i,\mathrm{bnd}}$

$$\frac{\partial n_i}{\partial \rho}|_{\rho=\rho_{\mathrm{bnd}}} = -\nabla n_{i,\mathrm{bnd}} \tag{2.41}$$
$$v(2) = 1 \tag{2.42}$$
$$u(2) = 0 \tag{2.43}$$
$$w(2) = -\nabla n_{i,\mathrm{bnd}} \tag{2.44}$$

type=3 (scale length) NI_BND_TYPE(2)=3; NI_BND(2,1)=L_{ni}

$$\frac{1}{(\partial \mathrm{ln} n_i/\partial \rho)}|_{\rho=\rho_{\mathrm{bnd}}} = -L_{\mathrm{ni}} \tag{2.45}$$
$$v(2) = 1 \tag{2.46}$$
$$u(2) = 1/L_{\mathrm{ni}} \tag{2.47}$$
$$w(2) = 0 \tag{2.48}$$

type=4 (flux) NI_BND_TYPE(2)=4; NI_BND(2,1)=$\Gamma_{i,\mathrm{bnd}}$

$$V' \left\langle |\nabla\rho|^2 \right\rangle \left(-D_i \frac{\partial n_i}{\partial \rho} + n_i\, V_i^{\mathrm{pinch}} \right)|_{\rho=\rho_{\mathrm{bnd}}} = \Gamma_{i,\mathrm{bnd}} \tag{2.49}$$
$$v(2) = -V' \left\langle |\nabla\rho|^2 \right\rangle D_i \tag{2.50}$$
$$u(2) = V' \left\langle |\nabla\rho|^2 \right\rangle V_i^{\mathrm{pinch}} \tag{2.51}$$
$$w(2) = \Gamma_{i,\mathrm{bnd}} \tag{2.52}$$

type=5 (generic) NI_BND_TYPE(2)=5; NI_BND(2,1)=v_{gen}; NI_BND(2,2)=u_{gen}; NI_BND(2,3)=w_{gen}

$$v_{\mathrm{gen}} \left.\frac{\partial n_i}{\partial \rho}\right|_{\mathrm{bnd}} + u_{\mathrm{gen}} n_{i,\mathrm{bnd}} = w_{\mathrm{gen}} \tag{2.53}$$
$$v(2) = v_{\mathrm{gen}} \tag{2.54}$$
$$u(2) = u_{\mathrm{gen}} \tag{2.55}$$
$$w(2) = w_{\mathrm{gen}} \tag{2.56}$$

*** extra output, ion flux contributing to ion energy transport**

$$\gamma_i = \sum_{i\mathrm{model}=1}^{n\mathrm{model}} c_{1,i\mathrm{model}} V' \left\langle |\nabla\rho|^2 \right\rangle \left(-D_{i,i\mathrm{model}} \frac{\partial n_i}{\partial \rho} + n_i\, V_{i,i\mathrm{model}}^{\mathrm{pinch}} \right) \tag{2.57}$$

2.4 Variables used in ETS (SUBROUTINE ION_DENSITY)

Variable	**TYPE%NAME used in ETS data flow**	**Internal name used in ION_DENSITY routine**	**Units**
τ	EVOLUTION%TAU	TAU	s
$\dot{B}_0$	GEOMETRY%BTPRIME	BTPRIME	T/s
B_0	GEOMETRY%BT	BT	T
ρ	GEOMETRY%RHO	RHO	m
V'	GEOMETRY%VPR	VPR	m^2
V'^{-}	EVOLUTION%VPRM	VPRM	m^2
$\left\langle \lvert\nabla\rho\rvert^2 \right\rangle$	GEOMETRY%G1	G1	--
n_i	PROFILES%NI	NI	m^{-3}
n_{i^-}	EVOLUTION%NI	NIM	m^{-3}
Γ_i	PROFILES%FLUX_NI	FLUX	1/s
γ_i	PROFILES%FLUX_NI_CONV	FLUX_NI_CONV	1/s
Γ_{Si}	PROFILES%INT_SOURCE_NI	INT_SOURCE	1/s
$S_{i,\mathrm{exp}}$	-	SI_EXP	$m^{-3}s^{-1}$
$S_{i,\mathrm{imp}}$	-	SI_IMP	1/s
$S_{i,\mathrm{isource},1}$	SOURCES%SI_EXP	-	$m^{-3}s^{-1}$
$S_{i,\mathrm{isource},2}$	SOURCES%SI_IMP	-	1/s
D_i	--	DIFF	m^2/s
V_i^{pinch}	--	VCONV	m/s
$D_{i,i\mathrm{model}}$	TRANSPORT%DIFF_NI	DIFF_MOD	m^2/s
$V_{i,i\mathrm{model}}^{\mathrm{pinch}}$	TRANSPORT%VCONV_NI	VCONV_MOD	m/s
$c_{1,i\mathrm{model}}$	TRANSPORT%C1	C1	--
a	SOLVER%A	A	--
b	SOLVER%B	B	--
c	SOLVER%C	C	--
d	SOLVER%D	D	--
e	SOLVER%E	E	--
f	SOLVER%F	F	--
g	SOLVER%G	G	--
h	SOLVER%H	H	--
v(1:2)	SOLVER%V	V	--
u(1:2)	SOLVER%U	U	--
w(1:2)	SOLVER%W	W	--

Y^{t-1}	SOLVER%YM	W	--
solution	SOLVER%Y	Y	--
derivative of solution	SOLVER%DY	DY	--

Functions	**Internal name used in ION_DENSITY routine**
$\frac{1}{\rho}\left(V'S_{i,\mathrm{exp}} + \frac{V'^{-} n_{i,\mathrm{interp}^{-}}}{\tau} - n_{i,\mathrm{interp}} V' \cdot \left(\frac{1}{\tau} + S_{i,\mathrm{imp}}\right)\right)$	FUN1
$\int_0^{\rho}\left(V'S_{i,\mathrm{exp}} + \frac{V'^{-} n_{i,\mathrm{interp}^{-}}}{\tau} - n_{i,\mathrm{interp}} V' \cdot \left(\frac{1}{\tau} + S_{i,\mathrm{imp}}\right)\right)\partial\rho$	INTFUN1

3 Quasi-neutrality condition (electron density)

outcome:

$n_e(\rho)$ — electron density,
$\Gamma_e(\rho)$ — electron flux,
$\gamma_e(\rho)$ — contribution to electron heat transport,
$Z_{\text{eff}}(\rho)$ — effective charge

electron density and flux are estimated from:

$$n_e = \sum_{\text{ion}} Z_{\text{ion}} \cdot n_{\text{ion}} + \sum_{\text{imp}} Z_{\text{imp}} \cdot n_{\text{imp}} \quad (3.1)$$

$$\Gamma_e = \sum_{\text{ion}} Z_{\text{ion}} \cdot \Gamma_{\text{ion}} + \sum_{\text{imp}} Z_{\text{imp}} \cdot \Gamma_{\text{imp}} \quad (3.2)$$

$$Z_{\text{eff}} = \frac{\sum_{\text{ion}} Z_{\text{ion}}^2 \cdot n_{\text{ion}} + \sum_{\text{imp}} Z_{\text{imp}}^2 \cdot n_{\text{imp}}}{n_e} \quad (3.3)$$

$$\gamma_e = \sum_{\text{ion}} Z_{\text{ion}} \cdot \gamma_{\text{ion}} \quad (3.4)$$

*second terms are optional, included if impurity density and flux are computed by the separate “IMPURITY” module

*indexes *ion* and *imp* correspond to particular ionization state of particular ion

3.1 Variables used in ETS (SUBROUTINE QUASI_NEUTRALITY)

Variable	**TYPE%NAME used in ETS data flow**	**Internal name used in QUASI_NEUTRALITY routine**	**Units**
ρ	GEOMETRY%RHO	RHO	m
n_e	PROFILES%NE	NE	m^{-3}
Γ_e	PROFILES%FLUX_NE	FLUX_NE	1/s
γ_e	PROFILES%FLUX_NE_CONV	FLUX_NE_CONV	1/s
Z_{eff}	PROFILES%ZEFF	ZEFF	--
Z_{ion}	PROFILES%ZION	ZION	--
Z_{ion}^2	PROFILES%ZION2	ZION2	--
n_{ion}	PROFILES%NI	NI	m^{-3}
Γ_{ion}	PROFILES%FLUX_NI	FLUX_NI	1/s
γ_{ion}	PROFILES%FLUX_NI_CONV	FLUX_NI_CONV	1/s
Z_{imp}	IMPURITY%ZIMP	ZIMP	--

Z^2_{imp}	IMPURITY%ZIMP2	ZIMP2	--
n_{imp}	IMPURITY%NZ	NZ	m^{-3}
Γ_{imp}	IMPURITY %FLUX_NZ	FLUX_NZ	1/s

4 Ion energy transport equation (1:nion)

outcome:

$T_i\left(\rho, i_{\text{ion}}\right)$	—	ion temperature,
$q_i\left(\rho, i_{\text{ion}}\right)$	—	ion conductive heat flux,
$T_i\left(\rho, i_{\text{ion}}\right)\cdot\gamma_i\left(\rho, i_{\text{ion}}\right)$	—	ion convective heat flux,
$H_i\left(\rho, i_{\text{ion}}\right)$	—	total ion heat flux,
$H_{i,\text{int}}\left(\rho, i_{\text{ion}}\right)$	—	integral of sources

Ion energy equation to be solved:

$$\frac{3}{2}\left(\frac{\partial}{\partial t}-\frac{\dot{B}_0}{2B_0}\cdot\frac{\partial}{\partial\rho}\rho\right)\left(n_iT_iV'^{\frac{5}{3}}\right)+V'^{\frac{2}{3}}\frac{\partial}{\partial\rho}\left(q_i+T_i\,\gamma_i\right)=V'^{\frac{5}{3}}\left[Q_{i,\text{exp}}-Q_{i,\text{imp}}\cdot T_i+Q_{\text{ei}}+Q_{\text{zi}}+Q_{\gamma i}\right] \tag{4.1}$$

where conductive and convective heat flux defined as:

$$q_i=V'\left\langle\left|\nabla\rho\right|^2\right\rangle\left[n_i\left(-\chi_i\frac{\partial T_i}{\partial\rho}+T_i\,V_{\text{Ti}}^{\text{pinch}}\right)\right] \tag{4.2}$$

$$T_i\,\gamma_i=T_i\sum_{i\text{model}=1}^{n\text{model}}c_{1,i\text{model}}\Gamma_{i,i\text{model}} \tag{4.3}$$

(γ_i is computed in ION_DENSITY routine), and partial particle fluxes are defined as:

$$\Gamma_{i,i\text{model}}=V'\left\langle\left|\nabla\rho\right|^2\right\rangle\left(-D_{i,i\text{model}}\frac{\partial n_i}{\partial\rho}+n_i\,V_{i,i\text{model}}^{\text{pinch}}\right) \tag{4.4}$$

Total heat flux:

$$H_i=q_i+T_i\,\gamma_i \tag{4.5}$$

Total transport coefficients are defined as a sum of individual contributions **(1:NMODEL)**:

$$\chi_i=\chi_{i,\text{an}}+\chi_{i,\text{NC}}+\chi_{i,\text{ext}}+...=\sum_{i\text{model}=1}^{n\text{model}}\chi_{i,i\text{model}} \tag{4.6}$$

$$V_{\text{Ti}}^{\text{pinch}}=V_{\text{Ti,an}}^{\text{pinch}}+V_{\text{Ti,NC}}^{\text{pinch}}+V_{\text{Ti,ext}}^{\text{pinch}}+...=\sum_{i\text{model}=1}^{n\text{model}}V_{\text{Ti},i\text{model}}^{\text{pinch}} \tag{4.7}$$

(*individual contributions,* $\chi_{i,i\text{model}}$ and $V_{\text{Ti},i\text{model}}^{\text{pinch}}$, *to* χ_i *and* $V_{\text{Ti}}^{\text{pinch}}$ *by various transport models should be provided by IMP#4 modules*)

and right hand side includes contributions **(1:NSOURCE)** in generic form from external sources (*computed by other modules, like ICRH, ECRH, NBI, neutrals and etc.*), in the form of explicit and implicit terms (*index* 1 *is used for explicit parts and index* 2 *for implicit*):

$$Q_{i,\text{exp}}=\sum_{\text{isource}=1}^{\text{nsource}}Q_{i,\text{isource},1} \tag{4.8}$$

$$Q_{i,\text{imp}}=\sum_{\text{isource}=1}^{\text{nsource}}Q_{i,\text{isource},2} \tag{4.9}$$

(*individual contributions by various sources should be provided by IMP#5 modules, accept for neutrals and pellets, being the IMP#3 responsibility*)

and internal sources (*computed by ETS, various energy exchange components*):

$$Q_{\mathrm{ei}} = \nu_{\mathrm{ei}} \left(T_e - T_i\right) \tag{4.10}$$

$$Q_{\mathrm{zi}} = \sum_z \nu_{\mathrm{zi}} \left(T_z - T_i\right) = \sum_z \nu_{\mathrm{zi}} T_z - T_i \cdot \sum_z \nu_{\mathrm{zi}} \tag{4.11}$$

$$Q_{\gamma i} = \sum_{i\mathrm{model}=1}^{n\mathrm{model}} Q_{\gamma i, i\mathrm{model}} \tag{4.12}$$

various collisions quantities (ν_{ei}, $\sum_z \nu_{\mathrm{zi}}$, $\nu_{\mathrm{ei}} T_e$ *and* $\sum_z \nu_{\mathrm{zi}} T_z$*) should be provided by stand alone* COLLISIONS *module*

flow terms $Q_{\gamma i, i\mathrm{model}}$ *should be provided by transport modules, IMP#4*

4.1 Boundary conditions

Following options to specify boundary conditions should be available with the transport solver

NOTE! Specified positive values for $\nabla T_{i,\mathrm{bnd}}$ ***and*** L_{Ti} ***correspond to "normal" profile with temperature decreasing towards the edge***

on axis $\rho = 0$:

$$\left.\frac{\partial T_i}{\partial \rho}\right|_{\rho=0} = 0 \tag{4.13}$$

$$V_{\mathrm{Ti}}^{\mathrm{pinch}} = 0 \tag{4.14}$$

at the edge $\rho = \rho_{\mathrm{bnd}}$:

type=1 (value)

$$\left. T_i \right|_{\rho=\rho_{\mathrm{bnd}}} = T_{i,\mathrm{bnd}} \tag{4.15}$$

type=2 (gradient)

$$\left.\frac{\partial T_i}{\partial \rho}\right|_{\rho=\rho_{\mathrm{bnd}}} = -\nabla T_{i,\mathrm{bnd}} \tag{4.16}$$

type=3 (scale length)

$$\left.\frac{1}{(\partial \ln T_i / \partial \rho)}\right|_{\rho=\rho_{\mathrm{bnd}}} = -L_{\mathrm{Ti}} \tag{4.17}$$

type=4 (flux)

$$\left.(q_i + T_i \gamma_i)\right|_{\rho=\rho_{\mathrm{bnd}}} = f_{\mathrm{Ti,bnd}} \tag{4.18}$$

type=5 (generic)

$$v_{\mathrm{gen}} \left.\frac{\partial T_i}{\partial \rho}\right|_{\mathrm{bnd}} + u_{\mathrm{gen}} T_{i,\mathrm{bnd}} = w_{\mathrm{gen}} \tag{4.19}$$

If equation 4.1 is not solved (TI_BND_TYPE=0), interpretative temperature should be specified and various flux components are calculated from the integral of sources:

$$\begin{aligned} T_i &= T_{i,\text{interp}} && (4.20)\\ T_i\gamma_i &= \gamma_i \cdot T_{i,\text{interp}} && (4.21)\\ H_i &= H_{i,\text{int}} && (4.22)\\ q_i &= H_{i,\text{int}} - \gamma_i \cdot T_{i,\text{interp}} && (4.23)\end{aligned}$$

where:

$$\begin{aligned} H_{i,\text{int}} = \; & \frac{3}{2} \cdot \frac{\dot{B}_0}{2B_0} \cdot \rho n_i V' \cdot T_{i,\text{interp}} \\ & + \int_0^\rho V' \left[\frac{3}{2} \frac{n_{i^-} T_{i,\text{interp}^-}}{\tau} \left(\frac{V'^-}{V'} \right)^{\frac{5}{3}} + Q_{i,\exp} + Q_{\gamma i} + \nu_{\text{ei}} T_e + \sum_z \nu_{\text{zi}} T_z \right] \partial\rho \\ & - \int_0^\rho V' \left[\frac{3}{2} \frac{n_i}{\tau} + Q_{i,\text{imp}} + \nu_{\text{ei}} + \sum_z \nu_{\text{zi}} - \rho n_i \cdot \frac{\dot{B}_0}{2B_0} \cdot \frac{\partial V'}{\partial\rho} \right] \cdot T_{i,\text{interp}} \cdot \partial\rho \end{aligned} \quad (4.24)$$

4.2 Translation of the ion energy equation to the generalized form with numerical coefficients

time discretization:

$$\begin{aligned} \frac{3}{2} \frac{n_i T_i V'^{\frac{5}{3}} - n_{i^-} T_{i^-} V'^{-\frac{5}{3}}}{\tau} - \frac{3}{2} \frac{\dot{B}_0}{2B_0} \cdot \frac{\partial}{\partial\rho} \left(\rho n_i T_i V'^{\frac{5}{3}} \right) + V'^{\frac{2}{3}} \frac{\partial}{\partial\rho} \left(q_i + T_i \gamma_i \right) = \\ = V'^{\frac{5}{3}} \left[Q_{i,\exp} - Q_{i,\text{imp}} \cdot T_i + Q_{\text{ei}} + Q_{\text{zi}} + Q_{\gamma i} \right] \end{aligned} \quad (4.25)$$

where τ is the time step, T_{i^-}, n_{i^-} and V'^- are taken from the previous time step

$$\begin{aligned} \frac{3}{2} \frac{n_i T_i V'^{\frac{5}{3}} - n_{i^-} T_{i^-} V'^{-\frac{5}{3}}}{\tau} + \rho n_i T_i V'^{\frac{5}{3}} \cdot \left[\frac{3}{2} \frac{\partial}{\partial\rho} \left(\frac{\dot{B}_0}{2B_0} \right) - \frac{\dot{B}_0}{2B_0} \frac{1}{V'} \cdot \frac{\partial V'}{\partial\rho} \right] + \\ + V'^{\frac{2}{3}} \frac{\partial}{\partial\rho} \left(q_i + T_i \gamma_i - \frac{3}{2} \cdot \frac{\dot{B}_0}{2B_0} \cdot \rho n_i T_i V' \right) = V'^{\frac{5}{3}} \left[Q_{i,\exp} - Q_{i,\text{imp}} \cdot T_i + Q_{\text{ei}} + Q_{\text{zi}} + Q_{\gamma i} \right] \end{aligned} \quad (4.26)$$

$$\begin{aligned} \frac{3}{2} \frac{n_i T_i V'^{\frac{5}{3}} - n_{i^-} T_{i^-} V'^{-\frac{5}{3}}}{\tau} + \rho n_i T_i V'^{\frac{5}{3}} \cdot \left[\frac{3}{2} \frac{\partial}{\partial\rho} \left(\frac{\dot{B}_0}{2B_0} \right) - \frac{\dot{B}_0}{2B_0} \frac{1}{V'} \cdot \frac{\partial V'}{\partial\rho} \right] + \\ + V'^{\frac{2}{3}} \frac{\partial}{\partial\rho} \left(V' \left\langle |\nabla\rho|^2 \right\rangle \left[n_i \left(-\chi_i \frac{\partial T_i}{\partial\rho} + T_i V_{\text{Ti}}^{\text{pinch}} \right) \right] + T_i \gamma_i - \frac{3}{2} \cdot \frac{\dot{B}_0}{2B_0} \cdot \rho n_i T_i V' \right) = \\ ?V'^{\frac{5}{3}} \left[Q_{i,\exp} - Q_{i,\text{imp}} \cdot T_i + Q_{\gamma i} + \nu_{\text{ei}} \left(T_e - T_i \right) + \sum_z \nu_{\text{zi}} \left(T_z - T_i \right) \right] \end{aligned} \quad (4.27)$$

$$\begin{aligned} \frac{3}{2} \frac{n_i T_i V'^{\frac{5}{3}} - n_{i^-} T_{i^-} V'^{-\frac{5}{3}}}{\tau} + \rho n_i T_i V'^{\frac{5}{3}} \cdot \left[\frac{3}{2} \frac{\partial}{\partial\rho} \left(\frac{\dot{B}_0}{2B_0} \right) - \frac{\dot{B}_0}{2B_0} \frac{1}{V'} \cdot \frac{\partial V'}{\partial\rho} \right] + \\ + V'^{\frac{2}{3}} \frac{\partial}{\partial\rho} \left(V' \left\langle |\nabla\rho|^2 \right\rangle \left[n_i \left(-\chi_i \frac{\partial T_i}{\partial\rho} + T_i V_{\text{Ti}}^{\text{pinch}} \right) \right] + T_i \gamma_i - \frac{3}{2} \cdot \frac{\dot{B}_0}{2B_0} \cdot \rho n_i T_i V' \right) = \\ ?V'^{\frac{5}{3}} \left[Q_{i,\exp} - Q_{i,\text{imp}} \cdot T_i + Q_{\gamma i} + \nu_{\text{ei}} \left(T_e - T_i \right) + \sum_z \nu_{\text{zi}} \left(T_z - T_i \right) \right] \end{aligned} \quad (4.28)$$

$$
\begin{aligned}
&\tfrac{3}{2}V'\frac{n_iT_i - n_{i-}T_{i-}\left(\frac{V'-}{V'}\right)^{\frac{5}{3}}}{\tau} + T_i n_i \rho V' \cdot \left[\tfrac{3}{2}\tfrac{\partial}{\partial\rho}\left(\tfrac{\dot{B}_0}{2B_0}\right) - \tfrac{\dot{B}_0}{2B_0}\tfrac{1}{V'}\cdot\tfrac{\partial V'}{\partial\rho}\right] + \\
&+\tfrac{\partial}{\partial\rho}\left(V'\left\langle|\nabla\rho|^2\right\rangle\left[n_i\left(-\chi_i\tfrac{\partial T_i}{\partial\rho} + T_i\ V_{\mathrm{Ti}}^{\mathrm{pinch}}\right)\right] + T_i\gamma_i - \tfrac{3}{2}\cdot\tfrac{\dot{B}_0}{2B_0}\cdot\rho n_i T_i V'\right) = \\
&?V'\left[Q_{i,\mathrm{exp}} - Q_{i,\mathrm{imp}}\cdot T_i + Q_{\gamma i} + \nu_{\mathrm{ei}}\left(T_e - T_i\right) + \sum_z \nu_{\mathrm{zi}}\left(T_z - T_i\right)\right]
\end{aligned}
\tag{4.29}
$$

$$
\begin{aligned}
&\tfrac{3}{2}V'\frac{n_iT_i - n_{i-}T_{i-}\left(\frac{V'-}{V'}\right)^{\frac{5}{3}}}{\tau} + \\
&+\tfrac{\partial}{\partial\rho}\left(-V'\left\langle|\nabla\rho|^2\right\rangle n_i\ \chi_i\tfrac{\partial T_i}{\partial\rho} + \left[V'\left\langle|\nabla\rho|^2\right\rangle n_i V_{\mathrm{Ti}}^{\mathrm{pinch}} + \gamma_i - \tfrac{3}{2}\cdot\tfrac{\dot{B}_0}{2B_0}\cdot\rho n_i V'\right]\cdot T_i\right) = \\
&?V'\left[Q_{i,\mathrm{exp}} + Q_{\gamma i} + \nu_{\mathrm{ei}}T_e + \sum_z \nu_{\mathrm{zi}}T_z\right] - \\
&-V'\left[Q_{i,\mathrm{imp}} + \nu_{\mathrm{ei}} + \sum_z \nu_{\mathrm{zi}} + \rho V' n_i\cdot\left(\tfrac{3}{2}\tfrac{\partial}{\partial\rho}\left(\tfrac{\dot{B}_0}{2B_0}\right) - \tfrac{\dot{B}_0}{2B_0}\tfrac{1}{V'}\cdot\tfrac{\partial V'}{\partial\rho}\right)\right]\cdot T_i
\end{aligned}
\tag{4.30}
$$

Generalized form of diffusion equation for the quantity used in the ETS:

$$
\frac{a(\rho)\cdot Y(\rho) - b(\rho)\cdot Y^{t-1}(\rho)}{h} + \frac{1}{c(\rho)}\frac{\partial}{\partial\rho}\left(-d(\rho)\cdot\frac{\partial Y(\rho)}{\partial\rho} + e(\rho)\cdot Y(\rho)\right) = f(\rho) - g(\rho)\cdot Y(\rho) \tag{4.31}
$$

definitions for numerical coefficients in ion energy equation:

$$
a(\rho) = \frac{3}{2}V' n_i \tag{4.32}
$$

$$
b(\rho) = \frac{3}{2}n_{i-}\left(\frac{V'^{-\frac{5}{3}}}{V'^{\frac{2}{3}}}\right) \tag{4.33}
$$

$$
Y^{t-1}(\rho) = T_i^{-} \tag{4.34}
$$

$$
c(\rho) = 1 \tag{4.35}
$$

$$
d(\rho) = V'\left\langle|\nabla\rho|^2\right\rangle n_i\ \chi_i \tag{4.36}
$$

$$
e(\rho) = V'\left\langle|\nabla\rho|^2\right\rangle n_i V_{\mathrm{Ti}}^{\mathrm{pinch}} + \gamma_i - \frac{3}{2}\cdot\frac{\dot{B}_0}{2B_0}\cdot\rho n_i V' \tag{4.37}
$$

$$
f(\rho) = V'\left[Q_{i,\mathrm{exp}} + Q_{\gamma i} + \nu_{\mathrm{ei}}T_e + \sum_z \nu_{\mathrm{zi}}T_z\right] \tag{4.38}
$$

$$
g(\rho) = V'\left[Q_{i,\mathrm{imp}} + \nu_{\mathrm{ei}} + \sum_z \nu_{\mathrm{zi}} - \rho V' n_i\cdot\frac{\dot{B}_0}{2B_0}\frac{1}{V'}\cdot\frac{\partial V'}{\partial\rho}\right] \tag{4.39}
$$

$$
h = \tau \tag{4.40}
$$

4.3 Boundary conditions

generalized form required by numerical solver:

$v\frac{\partial Y}{\partial\rho}|_{\mathrm{bnd}} + \mathrm{uY}_{\mathrm{bnd}} = w$, *where v(1:2), u(1:2), w(1:2) are coefficients required by the solver*

on axis $\rho = 0$:

$$\left.\frac{\partial T_i}{\partial \rho}\right|_{\rho=0} = 0 \tag{4.41}$$
$$v(1) = 1 \tag{4.42}$$
$$u(1) = 0 \tag{4.43}$$
$$w(1) = 0 \tag{4.44}$$
$$e(1) = -\frac{3}{2}\cdot\frac{\dot{B}_0}{2B_0}\cdot\rho n_i V' \tag{4.45}$$

at the edge $\rho = \rho_{\text{bnd}}$:

type=1 (value) TI_BND_TYPE(2)=1; TI_BND(2,1)=$T_{i,\text{bnd}}$

$$T_i|_{\rho=\rho_{\text{bnd}}} = T_{i,\text{bnd}} \tag{4.46}$$
$$v(2) = 0 \tag{4.47}$$
$$u(2) = 1 \tag{4.48}$$
$$w(2) = T_{i,\text{bnd}} \tag{4.49}$$

type=2 (gradient) TI_BND_TYPE(2)=2; TI_BND(2,1)=$\nabla T_{i,\text{bnd}}$

$$\left.\frac{\partial T_i}{\partial \rho}\right|_{\rho=\rho_{\text{bnd}}} = -\nabla T_{i,\text{bnd}} \tag{4.50}$$
$$v(2) = 1 \tag{4.51}$$
$$u(2) = 0 \tag{4.52}$$
$$w(2) = -\nabla T_{i,\text{bnd}} \tag{4.53}$$

type=3 (scale length) TI_BND_TYPE(2)=3; TI_BND(2,1)=L_{Ti}

$$\left.\frac{1}{(\partial \ln T_i/\partial \rho)}\right|_{\rho=\rho_{\text{bnd}}} = -L_{\text{Ti}} \tag{4.54}$$
$$v(2) = 1 \tag{4.55}$$
$$u(2) = 1/L_{\text{Ti}} \tag{4.56}$$
$$w(2) = 0 \tag{4.57}$$

type=4 (flux) TI_BND_TYPE(2)=4; TI_BND(2,1)=$f_{\text{Ti,bnd}}$

$$(q_i + T_i\gamma_i)|_{\rho=\rho_{\text{bnd}}} = f_{\text{Ti,bnd}} \tag{4.58}$$
$$v(2) = -n_i\chi_i V'\left\langle|\nabla\rho|^2\right\rangle \tag{4.59}$$
$$u(2) = n_i V_{\text{Ti}}^{\text{pinch}} V'\left\langle|\nabla\rho|^2\right\rangle + \gamma_i \tag{4.60}$$
$$w(2) = f_{\text{Ti,bnd}} \tag{4.61}$$

type=5 (generic) TI_BND_TYPE(2)=5; TI_BND(2,1)=v_{gen}; TI_BND(2,2)=u_{gen}; TI_BND(2,3)=w_{gen}

$$v_{\text{gen}}\left.\frac{\partial T_i}{\partial \rho}\right|_{\text{bnd}} + u_{\text{gen}} T_{i,\text{bnd}} = w_{\text{gen}} \tag{4.62}$$
$$v(2) = v_{\text{gen}} \tag{4.63}$$
$$u(2) = u_{\text{gen}} \tag{4.64}$$
$$w(2) = w_{\text{gen}} \tag{4.65}$$

4.4 Variables used in ETS (SUBROUTINE ION_TEMPERATURE)

Variable	TYPE%NAME used in ETS data flow	Internal name used in ION_TEMPERATURE routine	Units
τ	EVOLUTION%TAU	TAU	s
$\dot{B}_0$	GEOMETRY%BTPRIME	BTPRIME	T/s
B_0	GEOMETRY%BT	BT	T
ρ	GEOMETRY%RHO	RHO	m
V'	GEOMETRY%VPR	VPR	m^2
V'^{-}	EVOLUTION%VPRM	VPRM	m^2
$\frac{\partial V'}{\partial \rho}$	--	DVPR	m
$\left\langle \lvert\nabla\rho\rvert^2 \right\rangle$	GEOMETRY%G1	G1	--
T_i	PROFILES%TI	TI	eV
T_{i^-}	EVOLUTION%TI	TIM	eV
n_i	PROFILES%NI	NI	m^{-3}
n_{i^-}	EVOLUTION%NI	NIM	m^{-3}
γ_i	PROFILES%FLUX_NI_CONV	FLUX_NI	1/s
H_i	PROFILES%FLUX_TI	FLUX_TI	W
$T_i\gamma_i$	PROFILES%FLUX_TI_CONV	FLUX_TI_CONV	W
q_i	PROFILES%FLUX_TI_COND	FLUX_TI_COND	W
$H_{i,\mathrm{int}}$	PROFILES%INT_SOURCE_TI	INT_SOURCE	W
$Q_{i,\mathrm{exp}}$	--	QI_EXP	$\mathrm{eV\cdot m^{-3}s^{-1}}$
$Q_{i,\mathrm{imp}}$	--	QI_IMP	$\mathrm{m^{-3}s^{-1}}$
$Q_{i,\mathrm{isource},1}$	SOURCES%QI_EXP	--	$\mathrm{eV\cdot m^{-3}s^{-1}}$
$Q_{i,\mathrm{isource},2}$	SOURCES%QI_IMP	--	$\mathrm{m^{-3}s^{-1}}$
ν_{ei}	COLLISIONS%VEI	VEI	$\mathrm{m^{-3}s^{-1}}$
$\nu_{\mathrm{ei}}T_e$	COLLISIONS %QEI	QEI	$\mathrm{eV\cdot m^{-3}s^{-1}}$
$\sum_z \nu_{\mathrm{zi}}$	COLLISIONS %VZI	VZI	$\mathrm{m^{-3}s^{-1}}$
$\sum_z \nu_{\mathrm{zi}}T_z$	COLLISIONS %QZI	QZI	$\mathrm{eV\cdot m^{-3}s^{-1}}$
$Q_{\gamma i}$	--	QGI	$\mathrm{eV\cdot m^{-3}s^{-1}}$
$Q_{\gamma i,i\mathrm{model}}$	TRANSPORT%QGI	--	$\mathrm{eV\cdot m^{-3}s^{-1}}$
χ_i	--	DIFF	m^2/s
$V_{\mathrm{Ti}}^{\mathrm{pinch}}$	--	VCONV	m/s
$\chi_{i,i\mathrm{model}}$	TRANSPORT%DIFF_TI	--	m^2/s
$V_{\mathrm{Ti},i\mathrm{model}}^{\mathrm{pinch}}$	TRANSPORT%VCONV_TI	--	m/s

a	SOLVER%A	A	--
b	SOLVER%B	B	--
c	SOLVER%C	C	--
d	SOLVER%D	D	--
e	SOLVER%E	E	--
f	SOLVER%F	F	--
g	SOLVER%G	G	--
h	SOLVER%H	H	--
v(1:2)	SOLVER%V	V	--
u(1:2)	SOLVER%U	U	--
w(1:2)	SOLVER%W	W	--
Y^{t-1}	SOLVER%YM	W	--
solution	SOLVER%Y	Y	--
derivative of solution	SOLVER%DY	DY	--

Functions	**Internal name used in ION_TEMPERATURE routine**
$\frac{V}{\rho}'(\frac{3}{2}\frac{n_i - T_{i,\text{interp}} -}{\tau}\left(\frac{V'-}{V'}\right)^{\frac{5}{3}} + Q_{i,\text{exp}} + Q_{\gamma i} + \nu_{\text{ei}} T_e + \sum_z \nu_{\text{zi}} T_z - - \left[\frac{3}{2}\frac{n_i}{\tau} + Q_{i,\text{imp}} + \nu_{\text{ei}} + \sum_z \nu_{\text{zi}} - \rho n_i \cdot \frac{\dot{B}_0}{2B_0} \cdot \frac{\partial V'}{\partial \rho}\right] \cdot T_{i,\text{interp}})$	FUN1
$\int_0^\rho V' \left[\frac{3}{2}\frac{n_i - T_{i,\text{interp}} -}{\tau}\left(\frac{V'-}{V'}\right)^{\frac{5}{3}} + Q_{i,\text{exp}} + Q_{\gamma i} + \nu_{\text{ei}} T_e + \sum_z \nu_{\text{zi}} T_z\right] \partial\rho - -\int_0^\rho V' \left[\frac{3}{2}\frac{n_i}{\tau} + Q_{i,\text{imp}} + \nu_{\text{ei}} + \sum_z \nu_{\text{zi}} - \rho n_i \cdot \frac{\dot{B}_0}{2B_0} \cdot \frac{\partial V'}{\partial \rho}\right] \cdot T_{i,\text{interp}} \cdot \partial\rho$	INTFUN1

5 Electron energy transport equation

outcome:

$T_e(\rho)$	—	electron temperature,
$q_e(\rho)$	—	electron conductive heat flux,
$T_e(\rho)\cdot\gamma_e(\rho)$	—	electron convective heat flux,
$H_e(\rho)$	—	total electron heat flux,
$H_{e,\mathrm{int}}(\rho)$	—	integral of sources

electron energy equation to be solved:

$$\frac{3}{2}\left(\frac{\partial}{\partial t}-\frac{\dot{B}_0}{2B_0}\cdot\frac{\partial}{\partial\rho}\rho\right)\left(n_eT_eV'^{\frac{5}{3}}\right)+V'^{\frac{2}{3}}\frac{\partial}{\partial\rho}\left(q_e+T_e\gamma_e\right)=V'^{\frac{5}{3}}\left[Q_{e,\mathrm{exp}}-Q_{e,\mathrm{imp}}\cdot T_e+Q_{\mathrm{ie}}-Q_{\gamma i}\right] \quad (5.1)$$

where conductive and convective heat flux defined as:

$$q_e = V'\left\langle|\nabla\rho|^2\right\rangle\left[n_e\left(-\chi_e\frac{\partial T_e}{\partial\rho}+T_e\,V_{\mathrm{Te}}^{\mathrm{pinch}}\right)\right] \quad (5.2)$$

$$T_e\gamma_e = T_e\cdot\sum_i Z_i\cdot\gamma_i \quad (5.3)$$

Total transport coefficients are defined as a sum of individual contributions **(1:NMODEL)**:

$$\chi_e=\chi_{e,\mathrm{an}}+\chi_{e,\mathrm{NC}}+\chi_{e,\mathrm{ext}}+...=\sum_{i\mathrm{model}=1}^{n\mathrm{model}}\chi_{e,i\mathrm{model}} \quad (5.4)$$

$$V_{\mathrm{Te}}^{\mathrm{pinch}}=V_{\mathrm{Te,an}}^{\mathrm{pinch}}+V_{\mathrm{Te,NC}}^{\mathrm{pinch}}+V_{\mathrm{Te,ext}}^{\mathrm{pinch}}+...=\sum_{i\mathrm{model}=1}^{n\mathrm{model}}V_{\mathrm{Te},i\mathrm{model}}^{\mathrm{pinch}} \quad (5.5)$$

(*individual contributions,* $\chi_{e,i\mathrm{model}}$ and $V_{\mathrm{Te},i\mathrm{model}}^{\mathrm{pinch}}$, *to* χ_e *and* $V_{\mathrm{Te}}^{\mathrm{pinch}}$ *by various transport models should be provided by IMP#4 modules*)

and right hand side includes contributions **(1:NSOURCE)** in generic form from external sources (*computed by other modules, like ICRH, ECRH, NBI, neutrals and etc.*), in the form of explicit and implicit terms (*index* 1 *is used for explicit parts and index* 2 *for implicit*):

$$Q_{e,\mathrm{exp}} = \sum_{\mathrm{isource}=1}^{\mathrm{nsource}}Q_{e,\mathrm{isource},1}+Q_{\mathrm{OH}} \quad (5.6)$$

$$Q_{e,\mathrm{imp}} = \sum_{\mathrm{isource}=1}^{\mathrm{nsource}}Q_{e,\mathrm{isource},2} \quad (5.7)$$

(*individual contributions by various sources should be provided by IMP#5 modules, accept for neutrals and pellets, being the IMP#3 responsibility*)

and internal sources (*computed by ETS, various energy exchange components*):

$$Q_{\mathrm{ie}}=\sum_i\nu_{\mathrm{ei}}\left(T_i-T_e\right)=\sum_i\nu_{\mathrm{ei}}T_i-T_e\sum_i\nu_{\mathrm{ei}} \quad (5.8)$$

various collisions quantities ($\sum_i \nu_{\rm ei}$ and $\sum_i \nu_{\rm ei} T_i$) should be provided by stand alone COLLISIONS module

$$Q_{\gamma i} = \sum_{i\mathrm{model}=1}^{n\mathrm{model}} Q_{\gamma i, i\mathrm{model}} \tag{5.9}$$

flow terms $Q_{\gamma i, i\mathrm{model}}$ should be provided by transport modules, IMP#4

5.1 Boundary conditions

Following options to specify boundary conditions should be available with the transport solver

NOTE! Specified positive values for $\nabla T_{e,\rm bnd}$ and $L_{\rm Te}$ correspond to "normal" profile with temperature decreasing towards the edge

on axis $\rho = 0$:

$$\left.\frac{\partial T_e}{\partial \rho}\right|_{\rho=0} = 0 \tag{5.10}$$

at the edge $\rho = \rho_{\rm bnd}$:

type=1 (value)

$$T_e|_{\rho=\rho_{\rm bnd}} = T_{e,\rm bnd} \tag{5.11}$$

type=2 (gradient)

$$\left.\frac{\partial T_e}{\partial \rho}\right|_{\rho=\rho_{\rm bnd}} = -\nabla T_{e,\rm bnd} \tag{5.12}$$

type=3 (scale length)

$$\left.\frac{1}{(\partial \ln T_e / \partial \rho)}\right|_{\rho=\rho_{\rm bnd}} = -L_{\rm Te} \tag{5.13}$$

type=4 (flux)

$$(q_e + T_e \gamma_e)|_{\rho=\rho_{\rm bnd}} = f_{\rm Te,bnd} \tag{5.14}$$

type=5 (generic)

$$v_{\rm gen} \left.\frac{\partial T_e}{\partial \rho}\right|_{\rm bnd} + u_{\rm gen} T_{e,\rm bnd} = w_{\rm gen} \tag{5.15}$$

If equation 5.1 is not solved (TE_BND_TYPE=0), interpretative temperature should be specified and flux is assumed:

$$T_e = T_{e,\rm interp} \tag{5.16}$$

$$T_e \gamma_e = \gamma_e \cdot T_{e,\rm interp} \tag{5.17}$$

$$H_e = H_{e,\rm int} \tag{5.18}$$

$$q_e = H_{e,\rm int} - \gamma_e \cdot T_{e,\rm interp} \tag{5.19}$$

where:

$$\begin{gathered} H_{e,\rm int} = \tfrac{3}{2} \cdot \tfrac{\dot{B}_0}{2B_0} \cdot \rho n_e V' \cdot T_{e,\rm interp} + \\ + \int_0^\rho V' \left(\tfrac{3}{2} \tfrac{n_e - T_{e,\rm interp} -}{\tau} \left(\tfrac{V'-}{V'} \right)^{\frac{5}{3}} + Q_{e,\rm exp} + \sum_i \nu_{\rm ei} T_i - Q_{\gamma i} \right) \partial\rho - \\ - \int_0^\rho V' \left(-\tfrac{3}{2} \tfrac{n_e}{\tau} + Q_{e,\rm imp} + \sum_i \nu_{\rm ei} - \rho n_e \cdot \tfrac{\dot{B}_0}{2B_0} \tfrac{1}{V'} \cdot \tfrac{\partial V'}{\partial \rho} \right) \cdot T_{e,\rm interp} \partial\rho \end{gathered} \tag{5.20}$$

5.2 Translation of the electron energy equation to the generalized form with numerical coefficients

time discretization:

$$\frac{3}{2}\frac{n_eT_eV'^{\frac{5}{3}}-n_{e^-}T_{e^-}V'^{-\frac{5}{3}}}{\tau}-\frac{3}{2}\frac{\dot{B}_0}{2B_0}\cdot\frac{\partial}{\partial\rho}\left(\rho n_eT_eV'^{\frac{5}{3}}\right)+V'^{\frac{2}{3}}\frac{\partial}{\partial\rho}\left(q_e+T_e\gamma_e\right)= \\ =V'^{\frac{5}{3}}\left[Q_{e,\mathrm{exp}}-Q_{e,\mathrm{imp}}\cdot T_e+Q_{\mathrm{ie}}-Q_{\gamma i}\right] \quad (5.21)$$

where τ is the time step, T_{e^-}, n_{e^-} and V'^- are taken from the previous time step

$$\frac{3}{2}\frac{n_eT_eV'^{\frac{5}{3}}-n_{e^-}T_{e^-}V'^{-\frac{5}{3}}}{\tau}+\rho n_eT_eV'^{\frac{5}{3}}\cdot\left[\frac{3}{2}\frac{\partial}{\partial\rho}\left(\frac{\dot{B}_0}{2B_0}\right)-\frac{\dot{B}_0}{2B_0}\frac{1}{V'}\cdot\frac{\partial V'}{\partial\rho}\right]+ \\ +V'^{\frac{2}{3}}\frac{\partial}{\partial\rho}\left(q_e+T_e\gamma_e-\frac{3}{2}\cdot\frac{\dot{B}_0}{2B_0}\cdot\rho n_eT_eV'\right)=V'^{\frac{5}{3}}\left[Q_{e,\mathrm{exp}}-Q_{e,\mathrm{imp}}\cdot T_e+Q_{\mathrm{ie}}-Q_{\gamma i}\right] \quad (5.22)$$

$$\frac{3}{2}\frac{n_eT_eV'^{\frac{5}{3}}-n_{e^-}T_{e^-}V'^{-\frac{5}{3}}}{\tau}+\rho n_eT_eV'^{\frac{5}{3}}\cdot\left[\frac{3}{2}\frac{\partial}{\partial\rho}\left(\frac{\dot{B}_0}{2B_0}\right)-\frac{\dot{B}_0}{2B_0}\frac{1}{V'}\cdot\frac{\partial V'}{\partial\rho}\right]+ \\ +V'^{\frac{2}{3}}\frac{\partial}{\partial\rho}\left(V'\left\langle|\nabla\rho|^2\right\rangle\left[n_e\left(-\chi_e\frac{\partial T_e}{\partial\rho}+T_i\,V_{\mathrm{Te}}^{\mathrm{pinch}}\right)\right]+T_e\gamma_e-\frac{3}{2}\cdot\frac{\dot{B}_0}{2B_0}\cdot\rho n_eT_eV'\right)= \\ ?V'^{\frac{5}{3}}\left[Q_{e,\mathrm{exp}}-Q_{e,\mathrm{imp}}\cdot T_e+\sum_i\nu_{\mathrm{ei}}\left(T_i-T_e\right)-Q_{\gamma i}\right] \quad (5.23)$$

$$\frac{3}{2}V'\frac{n_eT_e-n_{e^-}T_{e^-}\left(\frac{V'^-}{V'}\right)^{\frac{5}{3}}}{\tau}+\frac{\partial}{\partial\rho}\left(V'\left\langle|\nabla\rho|^2\right\rangle\left[n_e\left(-\chi_e\frac{\partial T_e}{\partial\rho}+T_e\,V_{\mathrm{Te}}^{\mathrm{pinch}}\right)\right]+T_e\gamma_e-\frac{3}{2}\cdot\frac{\dot{B}_0}{2B_0}\cdot\rho n_eT_eV'\right)= \\ ?V'\left[Q_{e,\mathrm{exp}}+\sum_i\nu_{\mathrm{ei}}T_i-Q_{\gamma i}\right]-V'\left[Q_{e,\mathrm{imp}}+\sum_i\nu_{\mathrm{ei}}+\rho n_e\cdot\left(\frac{3}{2}\frac{\partial}{\partial\rho}\left(\frac{\dot{B}_0}{2B_0}\right)-\frac{\dot{B}_0}{2B_0}\frac{1}{V'}\cdot\frac{\partial V'}{\partial\rho}\right)\right]\cdot T_e \quad (5.24)$$

$$\frac{3}{2}V'\frac{n_eT_e-n_{e^-}T_{e^-}\left(\frac{V'^-}{V'}\right)^{\frac{5}{3}}}{\tau}+\frac{\partial}{\partial\rho}\left[-V'\left\langle|\nabla\rho|^2\right\rangle n_e\,\chi_e\frac{\partial T_e}{\partial\rho}+\left(V'\left\langle|\nabla\rho|^2\right\rangle n_e\,V_{\mathrm{Te}}^{\mathrm{pinch}}+\gamma_e-\frac{3}{2}\cdot\frac{\dot{B}_0}{2B_0}\cdot\rho n_eV'\right)\cdot T_e\right]= \\ ?V'\left[Q_{e,\mathrm{exp}}+\sum_i\nu_{\mathrm{ei}}T_i-Q_{\gamma i}\right]-V'\left[Q_{e,\mathrm{imp}}+\sum_i\nu_{\mathrm{ei}}+\rho n_e\cdot\left(\frac{3}{2}\frac{\partial}{\partial\rho}\left(\frac{\dot{B}_0}{2B_0}\right)-\frac{\dot{B}_0}{2B_0}\frac{1}{V'}\cdot\frac{\partial V'}{\partial\rho}\right)\right]\cdot T_e \quad (5.25)$$

Generalized form of diffusion equation for the quantity used in the ETS:

$$\frac{a(\rho)\cdot Y(\rho)-b(\rho)\cdot Y^{t-1}(\rho)}{h}+\frac{1}{c(\rho)}\frac{\partial}{\partial\rho}\left(-d(\rho)\cdot\frac{\partial Y(\rho)}{\partial\rho}+e(\rho)\cdot Y(\rho)\right)=f(\rho)-g(\rho)\cdot Y(\rho) \quad (5.26)$$

definitions for numerical coefficients in electron energy equation:

$$
\begin{aligned}
a(\rho) &= \frac{3}{2}V'n_e && (5.27)\\
b(\rho) &= \frac{3}{2}n_{e^-}\left(\frac{V'^{-\frac{5}{3}}}{V'^{\frac{2}{3}}}\right) && (5.28)\\
Y^{t-1}(\rho) &= T_e^- && (5.29)\\
c(\rho) &= 1 && (5.30)\\
d(\rho) &= V'\left\langle|\nabla\rho|^2\right\rangle n_e\,\chi_e && (5.31)\\
e(\rho) &= V'\left\langle|\nabla\rho|^2\right\rangle n_e\,V_{\mathrm{Te}}^{\mathrm{pinch}} + \gamma_e - \frac{3}{2}\cdot\frac{\dot{B}_0}{2B_0}\cdot\rho n_e V' && (5.32)\\
f(\rho) &= V'\left[Q_{e,\exp} + \sum_i \nu_{\mathrm{ei}}T_i - Q_{\gamma i}\right] && (5.33)\\
g(\rho) &= V'\left[Q_{e,\mathrm{imp}} + \sum_i \nu_{\mathrm{ei}} - \rho n_e\cdot\frac{\dot{B}_0}{2B_0}\frac{1}{V'}\cdot\frac{\partial V'}{\partial\rho}\right] && (5.34)\\
h &= \tau && (5.35)
\end{aligned}
$$

5.3 Boundary conditions

generalized form required by numerical solver:

$v\frac{\partial Y}{\partial\rho}|_{\mathrm{bnd}} + \mathrm{uY}_{\mathrm{bnd}} = w$, *where v(1:2), u(1:2), w(1:2) are coefficients required by the solver*

on axis $\rho = 0$:

$$
\begin{aligned}
\left.\frac{\partial T_e}{\partial\rho}\right|_{\rho=0} &= 0 && (5.36)\\
H_i &= 0 && (5.37)\\
v(1) &= 1 && (5.38)\\
u(1) &= 0 && (5.39)\\
w(1) &= 0 && (5.40)\\
e(1) &= -\frac{3}{2}\cdot\frac{\dot{B}_0}{2B_0}\cdot\rho n_e V' && (5.41)
\end{aligned}
$$

at the edge $\rho = \rho_{\mathrm{bnd}}$:

type=1 (value) TE_BND_TYPE(2)=1; TE_BND(2,1)=$T_{e,\mathrm{bnd}}$

$$
\begin{aligned}
T_e|_{\rho=\rho_{\mathrm{bnd}}} &= T_{e,\mathrm{bnd}} && (5.42)\\
v(2) &= 0 && (5.43)\\
u(2) &= 1 && (5.44)\\
w(2) &= T_{e,\mathrm{bnd}} && (5.45)
\end{aligned}
$$

type=2 (gradient) TE_BND_TYPE(2)=2; TE_BND(2,1)=$\nabla T_{e,\mathrm{bnd}}$

$$
\begin{aligned}
\left.\frac{\partial T_e}{\partial\rho}\right|_{\rho=\rho_{\mathrm{bnd}}} &= -\nabla T_{e,\mathrm{bnd}} && (5.46)\\
v(2) &= 1 && (5.47)\\
u(2) &= 0 && (5.48)\\
w(2) &= -\nabla T_{e,\mathrm{bnd}} && (5.49)
\end{aligned}
$$

type=3 (scale length) TE_BND_TYPE(2)=3; TE_BND(2,1)=L_{Te}

$$\left.\frac{1}{(\partial \ln T_e/\partial\rho)}\right|_{\rho=\rho_{\mathrm{bnd}}} = -L_{\mathrm{Te}} \tag{5.50}$$
$$v(2) = 1 \tag{5.51}$$
$$u(2) = 1/L_{\mathrm{Te}} \tag{5.52}$$
$$w(2) = 0 \tag{5.53}$$

type=4 (flux) TE_BND_TYPE(2)=4; TE_BND(2,1)=$f_{\mathrm{Te,bnd}}$

$$\left.(q_e + T_e\gamma_e)\right|_{\rho=\rho_{\mathrm{bnd}}} = f_{\mathrm{Te,bnd}} \tag{5.54}$$
$$v(2) = -n_e\chi_e V' \left\langle |\nabla\rho|^2 \right\rangle \tag{5.55}$$
$$u(2) = n_e V_{\mathrm{Te}}^{\mathrm{pinch}} V' \left\langle |\nabla\rho|^2 \right\rangle + \gamma_e \tag{5.56}$$
$$w(2) = f_{\mathrm{Te,bnd}} \tag{5.57}$$

type=5 (generic) TE_BND_TYPE(2)=5; TE_BND(2,1)=v_{gen}; TE_BND(2,2)=u_{gen}; TE_BND(2,3)=w_{gen}

$$v_{\mathrm{gen}} \left.\frac{\partial T_e}{\partial\rho}\right|_{\mathrm{bnd}} + u_{\mathrm{gen}} T_{e,\mathrm{bnd}} = w_{\mathrm{gen}} \tag{5.58}$$
$$v(2) = v_{\mathrm{gen}} \tag{5.59}$$
$$u(2) = u_{\mathrm{gen}} \tag{5.60}$$
$$w(2) = w_{\mathrm{gen}} \tag{5.61}$$

5.4 Variables used in ETS (SUBROUTINE ELECTRON_TEMPERATURE)

Variable	**TYPE%NAME used in ETS data flow**	**Internal name used in ELECTRON_TEMPERATURE routine**	**Units**
τ	EVOLUTION%TAU	TAU	s
$\dot{B}_0$	GEOMETRY%BTPRIME	BTPRIME	T/s
B_0	GEOMETRY%BT	BT	T
ρ	GEOMETRY%RHO	RHO	m
V'	GEOMETRY%VPR	VPR	m^2
V'^{-}	EVOLUTION%VPRM	VPRM	m^2
$\frac{\partial V'}{\partial \rho}$	--	DVPR	m
$\left\langle \|\nabla\rho\|^2 \right\rangle$	GEOMETRY%G1	G1	--
T_e	PROFILES%TE	TE	eV
T_{e^-}	EVOLUTION%TE	TEM	eV
n_e	PROFILES%NE	NE	m^{-3}
n_{e^-}	EVOLUTION%NE	NEM	m^{-3}

γ_e	PROFILES%FLUX_NE_CONV	FLUX_NE	1/s
H_e	PROFILES%FLUX_TE	FLUX_TE	W
$T_e\gamma_e$	PROFILES%FLUX_TE_CONV	FLUX_TE_CONV	W
q_e	PROFILES%FLUX_TE_COND	FLUX_TE_COND	W
$H_{e,\text{int}}$	PROFILES%INT_SOURCE_TE	INT_SOURCE	W
$Q_{e,\text{exp}}$	--	QE_EXP	eV·m^{-3}s^{-1}
$Q_{e,\text{imp}}$	--	QE_IMP	m^{-3}s^{-1}
$Q_{e,\text{isource},1}$	SOURCES%QE_EXP	--	eV·m^{-3}s^{-1}
$Q_{e,\text{isource},2}$	SOURCES%QE_IMP	--	m^{-3}s^{-1}
$\sum_i \nu_{\text{ei}}$	COLLISIONS%VIE	VIE	m^{-3}s^{-1}
$\sum_i \nu_{\text{ei}} T_i$	COLLISIONS %QIE	QIE	eV·m^{-3}s^{-1}
$Q_{\gamma i}$	--	QGI	eV·m^{-3}s^{-1}
$Q_{\gamma i,i\text{model}}$	TRANSPORT%QGI	--	eV·m^{-3}s^{-1}
χ_e	--	DIFF	m^2/s
$V_{\text{Te}}^{\text{pinch}}$	--	VCONV	m/s
$\chi_{e,i\text{model}}$	TRANSPORT%DIFF_TE	--	m^2/s
$V_{\text{Te},i\text{model}}^{\text{pinch}}$	TRANSPORT%VCONV_TE	--	m/s
a	SOLVER%A	A	--
b	SOLVER%B	B	--
c	SOLVER%C	C	--
d	SOLVER%D	D	--
e	SOLVER%E	E	--
f	SOLVER%F	F	--
g	SOLVER%G	G	--
h	SOLVER%H	H	--
v(1:2)	SOLVER%V	V	--
u(1:2)	SOLVER%U	U	--
w(1:2)	SOLVER%W	W	--
Y^{t-1}	SOLVER%YM	W	--
solution	SOLVER%Y	Y	--
derivative of solution	SOLVER%DY	DY	--

Functions	**Internal name used in ELECTRON_TEMPERATURE routine**

$\frac{V}{\rho}'(\frac{3}{2}\frac{n_{e}-T_{e,\text{interp}}-}{\tau}\left(\frac{V'-}{V'}\right)^{\frac{5}{3}}+Q_{e,\exp}+\sum_{i}\nu_{\text{ei}}T_{i}-Q_{\gamma i}$ $-\left[-\frac{3}{2}\frac{n_{e}}{\tau}Q_{e,\text{imp}}+\sum_{i}\nu_{\text{ei}}-\rho n_{e}\cdot\frac{\dot{B}_{0}}{2B_{0}}\frac{1}{V'}\cdot\frac{\partial V'}{\partial\rho}\right]\cdot T_{e,\text{interp}})$	FUN1
$\int_{0}^{\rho}V'\left(\frac{3}{2}\frac{n_{e}-T_{e,\text{interp}}-}{\tau}\left(\frac{V'-}{V'}\right)^{\frac{5}{3}}+Q_{e,\exp}+\sum_{i}\nu_{\text{ei}}T_{i}-Q_{\gamma i}\right)\partial\rho-$ $-\int_{0}^{\rho}V'\left(-\frac{3}{2}\frac{n_{e}}{\tau}Q_{e,\text{imp}}+\sum_{i}\nu_{\text{ei}}-\rho n_{e}\cdot\frac{\dot{B}_{0}}{2B_{0}}\frac{1}{V'}\cdot\frac{\partial V'}{\partial\rho}\right)\cdot T_{e,\text{interp}}\partial\rho$	INTFUN1

6 Rotation transport equation (1:nion)

outcome:

$u_{i,\varphi}(\rho, i_{\text{ion}})$ — ion toroidal rotation velocity,
$\omega_{i,\varphi}(\rho, i_{\text{ion}})$ — ion angular toroidal velocity,
$M_i(\rho, i_{\text{ion}})$ - ion toroidal momentum,
$\Phi_i(\rho, i_{\text{ion}})$ — ion toroidal momentum flux,
$\Phi_{i,\text{conv}}(\rho, i_{\text{ion}})$ — convective component of ion toroidal momentum flux,
$\Phi_{i,\text{cond}}(\rho, i_{\text{ion}})$ — conductive component of ion toroidal momentum flux,
$M_{\text{tot}}(\rho)$ — total ion toroidal momentum,
$\Phi_{\text{tot}}(\rho)$ — total ion toroidal momentum flux

equation for toroidal rotation velocity to be solved:

(terms with electron mass are neglected)

$$\left(\frac{\partial}{\partial t} - \frac{\dot{B}_0}{2B_0} \cdot \frac{\partial}{\partial \rho}\rho\right)\left(V' \ \langle R\rangle m_i n_i u_{i,\varphi}\right) + \frac{\partial}{\partial \rho}\Phi_i = V' \ \left(U_{i,\varphi,\text{exp}} - U_{i,\varphi,\text{imp}} \cdot u_{i,\varphi} + U_{\text{zi},\varphi}\right) \tag{6.1}$$

where total flux defined as:

$$\Phi_i = V' \left\langle |\nabla\rho|^2 \right\rangle m_i n_i \langle R\rangle \cdot \left(- \chi_{u\varphi,i} \frac{\partial u_{i,\varphi}}{\partial \rho} + u_{i,\varphi} V^{\text{pinch}}_{u\varphi,i}\right) + m_i \langle R\rangle u_{i,\varphi}\Gamma_i \tag{6.2}$$

with convective and conductive components:

$$\Phi_{i,\text{conv}} = m_i \langle R\rangle u_{i,\varphi}\Gamma_i \tag{6.3}$$

$$\Phi_{i,\text{cond}} = V' \left\langle |\nabla\rho|^2 \right\rangle m_i n_i \langle R\rangle \cdot \left(- \chi_{u\varphi,i} \frac{\partial u_{i,\varphi}}{\partial \rho} + u_{i,\varphi} V^{\text{pinch}}_{u\varphi,i}\right) \tag{6.4}$$

where total transport coefficients are defined as a sum of individual contributions **(1:NMODEL)**:

$$\chi_{u\varphi,i} = \chi_{u\varphi,\text{an}} + \chi_{u\varphi,\text{NC}} + \chi_{u\varphi,\text{ext}} + ... = \sum_{i\text{model}=1}^{n\text{model}} \chi_{u\varphi,i\text{model}} \tag{6.5}$$

$$V^{\text{pinch}}_{u\varphi} = V^{\text{pinch}}_{u\varphi,\text{an}} + V^{\text{pinch}}_{u\varphi,\text{NC}} + V^{\text{pinch}}_{u\varphi,\text{ext}} + ... = \sum_{i\text{model}=1}^{n\text{model}} V^{\text{pinch}}_{u\varphi,i\text{model}} \tag{6.6}$$

(*individual contributions,* $\chi_{u\varphi,i\text{model}}$ and $V^{\text{pinch}}_{u\varphi,i\text{model}}$, *to* $\chi_{u\varphi}$ *and* $V^{\text{pinch}}_{u\varphi}$ *by various transport models should be provided by IMP#4 modules*)

and right hand side includes contributions **(1:NSOURCE)** in generic form from external sources (*computed by other modules, like ICRH, ECRH, NBI, neutrals and etc.*), in the form of explicit and implicit terms (*index* 1 *is used for explicit parts and index* 2 *for implicit*):

$$U_{i,\varphi,\text{exp}} = \sum_{\text{isource}=1}^{\text{nsource}} U_{i,\varphi,\text{isource},1} \tag{6.7}$$

$$U_{i,\varphi,\text{imp}} = \sum_{\text{isource}=1}^{\text{nsource}} U_{i,\varphi,\text{isource},2} \tag{6.8}$$

(*individual contributions by various sources should be provided by IMP#5 modules, accept for neutrals and pellets, being the IMP#3 responsibility*)

momentum exchange with other ion components:

$$U_{\mathrm{zi},\varphi} = \langle R \rangle \sum_z \frac{m_{\mathrm{zi}} n_z}{\tau_{\mathrm{zi}}} \left(u_{z,\varphi} - u_{i,\varphi} \right) \tag{6.9}$$

various collisions quantities ($\langle R \rangle \sum\limits_z \frac{m_{\mathrm{zi}} n_z}{\tau_{\mathrm{zi}}}$ and $\langle R \rangle \sum\limits_z \frac{m_{\mathrm{zi}} n_z u_{z,\varphi}}{\tau_{\mathrm{zi}}}$) should be provided by stand alone COLLISIONS module

Other output quantities:

total toroidal momentum:

$$M_i = m_i n_i \langle R \rangle u_{i,\varphi} \tag{6.10}$$

angular velocity for plasma component i:

$$\omega_{i,\varphi} = \frac{u_{i,\varphi}}{\langle R \rangle} \tag{6.11}$$

total momentum and flux are defined as:

$$M_{\mathrm{tot}} = \sum_i M_i \tag{6.12}$$

$$\Phi_{\mathrm{tot}} = \sum_i \Phi_i \tag{6.13}$$

6.1 Boundary conditions

Following options to specify boundary conditions should be available with the transport solver

NOTE! Specified positive values for $\nabla u_{\varphi,\mathrm{bnd}}$ and $L_{u\varphi}$ correspond to "normal" profile with toroidal rotation velosity decreasing towards the edge

on axis $\rho = 0$:

$$\left. \frac{\partial u_{i,\varphi}}{\partial \rho} \right|_{\rho=0} = 0 \tag{6.14}$$

at the edge $\rho = \rho_{\mathrm{bnd}}$:

type=1 (value)

$$\left. u_{i,\varphi} \right|_{\rho=\rho_{\mathrm{bnd}}} = u_{\varphi,\mathrm{bnd}} \tag{6.15}$$

type=2 (gradient)

$$\left. \frac{\partial u_{i,\varphi}}{\partial \rho} \right|_{\rho=\rho_{\mathrm{bnd}}} = -\nabla u_{\varphi,\mathrm{bnd}} \tag{6.16}$$

type=3 (scale length)

$$\left. \frac{1}{(\partial \ln u_{i,\varphi} / \partial \rho)} \right|_{\rho=\rho_{\mathrm{bnd}}} = -L_{u\varphi} \tag{6.17}$$

type=4 (flux)

$$\left. \Phi \right|_{\rho=\rho_{\mathrm{bnd}}} = f_{u\varphi,\mathrm{bnd}} \tag{6.18}$$

type=5 (generic)

$$v_{\text{gen}} \left. \frac{\partial u_{i,\varphi,\text{bnd}}}{\partial \rho} \right|_{\text{bnd}} + u_{\text{gen}} u_{i,\varphi,\text{bnd}} = w_{\text{gen}} \tag{6.19}$$

If equation 6.1 is not solved (VTOR_BND_TYPE=0), interpretative toroidal velocity should be specified, momentum and flux are assumed:

$$\begin{aligned}
u_{i,\varphi} &= u_{i,\varphi,\text{interp}} && (6.20)\\
\omega_{i,\varphi} &= \frac{u_{i,\varphi,\text{interp}}}{\langle R \rangle} && (6.21)\\
M_i &= m_i n_i \langle R \rangle u_{i,\varphi,\text{interp}} && (6.22)\\
\Phi_{i,\text{cond}} &= \Phi_{i,\text{int}} - m_i \langle R \rangle \Gamma_i u_{i,\varphi,\text{interp}} && (6.23)\\
\Phi_i &= \Phi_{i,\text{int}} && (6.24)\\
\Phi_{i,\text{conv}} &= m_i \langle R \rangle \Gamma_i u_{i,\varphi,\text{interp}} && (6.25)\\
M_{\text{tot}} &= \sum_i M_i && (6.26)\\
\Phi_{\text{tot}} &= \sum_i \Phi_i && (6.27)
\end{aligned}$$

where:

$$\begin{gathered}
\Phi_{i,\text{int}} = V' \ \langle R \rangle \cdot \tfrac{\dot{B}_0}{2B_0} \cdot \rho m_i n_i u_{i,\varphi,\text{interp}} + \\
+ \int_0^\rho V' \left(U_{i,\varphi,\text{exp}} + \langle R \rangle \sum_z \tfrac{m_{\text{z}i} n_z u_{z,\varphi}}{\tau_{\text{z}i}} + \tfrac{\langle R \rangle^- m_i n_{i-} u^-_{i,\varphi,\text{interp}}}{\tau} \left(\tfrac{V'^-}{V'} \right) \right) \partial\rho \\
- \int_0^\rho V' \left(U_{i,\varphi,\text{imp}} - \langle R \rangle \tfrac{m_i n_i}{\tau} - \langle R \rangle \sum_z \tfrac{m_{\text{z}i} n_z}{\tau_{\text{z}i}} \right) u_{i,\varphi,\text{interp}} \partial\rho
\end{gathered} \tag{6.28}$$

6.2 Translation of the rotation equation to the generalized form with numerical coefficients

time discretization:

$$\frac{\partial}{\partial t} \left(V' m_i n_i \langle R \rangle u_{i,\varphi} \right) = m_i \frac{V' n_i \langle R \rangle u_{i,\varphi} - V'^- n_{i-} \langle R \rangle^- u_{i,\varphi^-}}{\tau} \tag{6.29}$$

where V'^-, n_{i-}, $\langle R \rangle^-$ and u_{i,φ^-} are taken at the previous time step;

$$\begin{gathered}
m_i \tfrac{V' n_i \langle R \rangle u_{i,\varphi} - V'^- n_{i-} \langle R \rangle^- u_{i,\varphi^-}}{\tau} - \left(\tfrac{\dot{B}_0}{2B_0} \cdot \tfrac{\partial}{\partial \rho} \rho \right) \left(V' \ \langle R \rangle m_i n_i u_{i,\varphi} \right) + \\
+ \tfrac{\partial}{\partial \rho} \left[V' \left\langle |\nabla\rho|^2 \right\rangle m_i n_i \langle R \rangle \cdot \left(- \chi_{u\varphi,i} \tfrac{\partial u_{i,\varphi}}{\partial \rho} + u_{i,\varphi} V^{\text{pinch}}_{u\varphi,i} \right) + m_i \langle R \rangle u_{i,\varphi} \Gamma_i \right] = \\
?V' \left(U_{i,\varphi,\text{exp}} - U_{i,\varphi,\text{imp}} \cdot u_{i,\varphi} + \langle R \rangle \sum_z \tfrac{m_{\text{z}i} n_z}{\tau_{\text{z}i}} \left(u_{z,\varphi} - u_{i,\varphi} \right) \right)
\end{gathered} \tag{6.30}$$

$$\begin{gathered}
m_i \tfrac{V' n_i \langle R \rangle u_{i,\varphi} - V'^- n_{i-} \langle R \rangle^- u_{i,\varphi^-}}{\tau} + V' m_i n_i \langle R \rangle u_{i,\varphi} \cdot \rho \tfrac{\partial}{\partial \rho} \left(\tfrac{\dot{B}_0}{2B_0} \right) + \\
+ \tfrac{\partial}{\partial \rho} \left[V' \left\langle |\nabla\rho|^2 \right\rangle m_i n_i \langle R \rangle \cdot \left(- \chi_{u\varphi,i} \tfrac{\partial u_{i,\varphi}}{\partial \rho} + u_{i,\varphi} V^{\text{pinch}}_{u\varphi,i} \right) + m_i \langle R \rangle u_{i,\varphi} \Gamma_i - \tfrac{\dot{B}_0}{2B_0} \cdot \left(\rho V' m_i n_i \langle R \rangle u_{i,\varphi} \right) \right] = \\
?V' \left(U_{i,\varphi,\text{exp}} - U_{i,\varphi,\text{imp}} \cdot u_{i,\varphi} + \langle R \rangle \sum_z \tfrac{m_{\text{z}i} n_z}{\tau_{\text{z}i}} \left(u_{z,\varphi} - u_{i,\varphi} \right) \right)
\end{gathered} \tag{6.31}$$

$$m_i \frac{V' n_i \langle R\rangle u_{i,\varphi} - V'^- n_{i^-} \langle R\rangle^- u_{i,\varphi^-}}{\tau} +$$

$$+\frac{\partial}{\partial\rho}\left[-V'\left\langle|\nabla\rho|^2\right\rangle m_i n_i \langle R\rangle \;\chi_{u\varphi,i}\frac{\partial u_{i,\varphi}}{\partial\rho} + \left(V'\left\langle|\nabla\rho|^2\right\rangle m_i n_i \langle R\rangle V^{\text{pinch}}_{u\varphi,i} + m_i\langle R\rangle\Gamma_i - \frac{\dot{B}_0}{2B_0}\cdot\left(\rho V'\langle R\rangle m_i n_i\right)\right)\cdot u_{i,\varphi}\right] =$$
$$?V'\left(U_{i,\varphi,\text{exp}} + \langle R\rangle\sum_z \frac{m_{\text{zi}} n_z u_{z,\varphi}}{\tau_{\text{zi}}}\right) - V'\left(U_{i,\varphi,\text{imp}}\cdot + \langle R\rangle\sum_z\frac{m_{\text{zi}}n_z}{\tau_{\text{zi}}} + \langle R\rangle m_i n_i\cdot\rho\frac{\partial}{\partial\rho}\left(\frac{\dot{B}_0}{2B_0}\right)\right)\cdot u_{i,\varphi} \tag{6.32}$$

Generalized form of diffusion equation for the quantity used in the ETS:

$$\frac{a(\rho)\cdot Y(\rho) - b(\rho)\cdot Y^{t-1}(\rho)}{h} + \frac{1}{c(\rho)}\frac{\partial}{\partial\rho}\left(-d(\rho)\cdot\frac{\partial Y(\rho)}{\partial\rho} + e(\rho)\cdot Y(\rho)\right) = f(\rho) - g(\rho)\cdot Y(\rho) \tag{6.33}$$

definitions for numerical coefficients in rotation equation:

$$a(\rho) = V'\langle R\rangle m_i n_i \tag{6.34}$$
$$b(\rho) = V'^-\langle R\rangle^- m_i n_{i^-} \tag{6.35}$$
$$Y^{t-1}(\rho) = u_{i,\varphi^-} \tag{6.36}$$
$$c(\rho) = 1 \tag{6.37}$$
$$d(\rho) = V'\left\langle|\nabla\rho|^2\right\rangle m_i n_i\langle R\rangle\;\chi_{u\varphi,i} \tag{6.38}$$
$$e(\rho) = V'\left\langle|\nabla\rho|^2\right\rangle m_i n_i\langle R\rangle V^{\text{pinch}}_{u\varphi,i} + m_i\langle R\rangle\Gamma_i - \frac{\dot{B}_0}{2B_0}\cdot\left(\rho V'\langle R\rangle m_i n_i\right) \tag{6.39}$$
$$f(\rho) = V'\left(U_{i,\varphi,\text{exp}} + \langle R\rangle\sum_z\frac{m_{\text{zi}}n_z u_{z,\varphi}}{\tau_{\text{zi}}}\right) \tag{6.40}$$
$$g(\rho) = V'\left[U_{i,\varphi,\text{imp}} + \langle R\rangle\sum_z\frac{m_{\text{zi}}n_z}{\tau_{\text{zi}}}\right] \tag{6.41}$$
$$h = \tau \tag{6.42}$$

6.3 Boundary conditions

generalized form required by numerical solver:

$v\frac{\partial Y}{\partial\rho}|_{\text{bnd}} + \text{uY}_{\text{bnd}} = w$, *where v(1:2), u(1:2), w(1:2) are coefficients required by the solver*

on axis $\rho = 0$:

$$\left.\frac{\partial u_{i,\varphi}}{\partial\rho}\right|_{\rho=0} = 0 \tag{6.43}$$
$$v(1) = 1 \tag{6.44}$$
$$u(1) = 0 \tag{6.45}$$
$$w(1) = 0 \tag{6.46}$$
$$e(1) = -\frac{\dot{B}_0}{2B_0}\cdot\left(\rho V'\langle R\rangle m_i n_i\right) \tag{6.47}$$

at the edge $\rho = \rho_{\text{bnd}}$:

type=1 (value) VTOR_BND_TYPE(2)=1; VTOR_BND(2,1)=$u_{\varphi,\text{bnd}}$

$$
\begin{aligned}
u_{i,\varphi}\big|_{\rho=\rho_{\text{bnd}}} &= u_{\varphi,\text{bnd}} &(6.48)\\
v\,(2) &= 0 &(6.49)\\
u\,(2) &= 1 &(6.50)\\
w\,(2) &= u_{\varphi,\text{bnd}} &(6.51)
\end{aligned}
$$

type=2 (gradient) VTOR_BND_TYPE(2)=2; VTOR_BND(2,1)=$\nabla u_{\varphi,\text{bnd}}$

$$
\begin{aligned}
\left.\frac{\partial u_{i,\varphi}}{\partial \rho}\right|_{\rho=\rho_{\text{bnd}}} &= -\nabla u_{\varphi,\text{bnd}} &(6.52)\\
v\,(2) &= 1 &(6.53)\\
u\,(2) &= 0 &(6.54)\\
w\,(2) &= -\nabla u_{\varphi,\text{bnd}} &(6.55)
\end{aligned}
$$

type=3 (scale length) VTOR_BND_TYPE(2)=3; VTOR_BND(2,1)=$L_{u\varphi}$

$$
\begin{aligned}
\left.\frac{1}{(\partial \ln u_{i,\varphi}/\partial \rho)}\right|_{\rho=\rho_{\text{bnd}}} &= -L_{u\varphi} &(6.56)\\
v\,(2) &= 1 &(6.57)\\
u\,(2) &= \frac{1}{L_{u\varphi}} &(6.58)\\
w\,(2) &= 0 &(6.59)
\end{aligned}
$$

type=4 (flux) VTOR_BND_TYPE(2)=4; VTOR_BND(2,1)=$f_{u\varphi,\text{bnd}}$

$$
\begin{aligned}
\Phi\big|_{\rho=\rho_{\text{bnd}}} &= f_{u\varphi,\text{bnd}} &(6.60)\\
\Phi_i &= V'\left\langle|\nabla\rho|^2\right\rangle m_i n_i \langle R\rangle \cdot \left(-\,\chi_{u\varphi,i}\frac{\partial u_{i,\varphi}}{\partial\rho} + u_{i,\varphi}V^{\text{pinch}}_{u\varphi,i}\right) + m_i\langle R\rangle u_{i,\varphi}\Gamma_i &(6.61)\\
v\,(2) &= -\,V'\left\langle|\nabla\rho|^2\right\rangle \langle R\rangle m_i n_i \chi_{u\varphi,i} &(6.62)\\
u\,(2) &= V'\left\langle|\nabla\rho|^2\right\rangle \langle R\rangle m_i n_i V^{\text{pinch}}_{u\varphi,i} + m_i\langle R\rangle \Gamma_i &(6.63)\\
w\,(2) &= f_{u\varphi,\text{bnd}} &(6.64)
\end{aligned}
$$

type=5 (generic) VTOR_BND_TYPE(2)=5; VTOR_BND(2,1)=v_{gen}; VTOR_BND(2,2)=u_{gen}; VTOR_BND(2,3)=w_{gen}

$$
\begin{aligned}
v_{\text{gen}}\left.\frac{\partial u_{i,\varphi,\text{bnd}}}{\partial\rho}\right|_{\text{bnd}} + u_{\text{gen}}u_{i,\varphi,\text{bnd}} &= w_{\text{gen}} &(6.65)\\
v\,(2) &= v_{\text{gen}} &(6.66)\\
u\,(2) &= u_{\text{gen}} &(6.67)\\
w\,(2) &= w_{\text{gen}} &(6.68)
\end{aligned}
$$

6.4 Variables used in ETS (SUBROUTINE ROTATION)

Variable	TYPE%NAME used in ETS data flow	Internal name used in ROTATION routine	Units

τ	EVOLUTION%TAU	TAU	s
$\dot{B}_0$	GEOMETRY%BTPRIME	BTPRIME	T/s
B_0	GEOMETRY%BT	BT	T
ρ	GEOMETRY%RHO	RHO	m
V'	GEOMETRY%VPR	VPR	m^2
V'^-	EVOLUTION%VPRM	VPRM	m^2
$\left\langle \lvert\nabla\rho\rvert^2 \right\rangle$	GEOMETRY%G1	G1	--
$\langle R \rangle$	GEOMETRY%G2	G2	m
$\langle R \rangle^-$	EVOLUTION%G2M	G2M	m
$u_{i,\varphi}$	PROFILES%VTOR	VTOR	m/s
u_{i,φ^-}	EVOLUTION%VTORM	VTORM	m/s
$\omega_{i,\varphi}$	PROFILES%WTOR	WTOR	1/s
M_i	PROFILES%MTOR	MTOR	kg*m/s
M_{tot}	PROFILES% MTOR_TOT	MTOR_TOT	kg*m/s
n_i	PROFILES%NI	NI	m^{-3}
n_{i^-}	EVOLUTION%NI	NIM	m^{-3}
Φ_i	PROFILES%FLUX_MTOR	FLUX_MTOR	$kg*m^2/s^2$
$\Phi_{i,\mathrm{conv}}$	PROFILES%FLUX_MTOR_CONV	FLUX_MTOR_CONV	$kg*m^2/s^2$
$\Phi_{i,\mathrm{cond}}$	PROFILES% FLUX_MTOR_COND	FLUX_MTOR_COND	$kg*m^2/s^2$
$\Phi_{i,\mathrm{int}}$	PROFILES% INT_SOURCE_MTOR	INT_SOURCE	$kg*m^2/s^2$
Φ_{tot}	PROFILES% FLUX_MTOR_TOT	FLUX_MTOR_TOT	$kg*m^2/s^2$
$U_{i,\varphi,\mathrm{exp}}$	--	UI_EXP	$kg/m/s^2$
$U_{i,\varphi,\mathrm{imp}}$	--	UI_IMP	$kg/m^2/s$
$U_{i,\varphi,\mathrm{isource},1}$	SOURCES%UI_EXP	--	$kg/m/s^2$
$U_{i,\varphi,\mathrm{isource},2}$	SOURCES%UI_IMP	--	$kg/m^2/s$
$\langle R \rangle \sum_z \frac{m_{\mathrm{zi}} n_z}{\tau_{\mathrm{zi}}}$	COLLISIONS %WZI	WZI	$kg/m^2/s$
$\langle R \rangle \sum_z \frac{m_{\mathrm{zi}} n_z u_{z,\varphi}}{\tau_{\mathrm{zi}}}$	COLLISIONS %UZI	UZI	$kg/m/s^2$
$\chi_{u\varphi,i}$	--	DIFF	m^2/s
$V_{u\varphi,i}^{\mathrm{pinch}}$	--	VCONV	m/s
$\chi_{u\varphi,i,\mathrm{imodel}}$	TRANSPORT%DIFF_VTOR	--	m^2/s
$V_{u\varphi,i,\mathrm{imodel}}^{\mathrm{pinch}}$	TRANSPORT%VCONV_VTOR	--	m/s
a	SOLVER%A	A	--

b	SOLVER%B	B	--
c	SOLVER%C	C	--
d	SOLVER%D	D	--
e	SOLVER%E	E	--
f	SOLVER%F	F	--
g	SOLVER%G	G	--
h	SOLVER%H	H	--
v(1:2)	SOLVER%V	V	--
u(1:2)	SOLVER%U	U	--
w(1:2)	SOLVER%W	W	--
Y^{t-1}	SOLVER%YM	W	--
solution	SOLVER%Y	Y	--
derivative of solution	SOLVER%DY	DY	--

Functions	**Internal name used in ROTATION routine**
$\frac{V'}{\rho}(U_{i,\varphi,\mathrm{exp}} + \langle R\rangle \sum_z \frac{m_{\mathrm{zi}} n_z u_{z,\varphi}}{\tau_{\mathrm{zi}}} + \frac{\langle R\rangle^- m_i n_{i-} u^-_{i,\varphi,\mathrm{interp}}}{\tau} \left(\frac{V'^-}{V'}\right) - \left(U_{i,\varphi,\mathrm{imp}} - \langle R\rangle \frac{m_i n_i}{\tau} - \langle R\rangle \sum_z \frac{m_{\mathrm{zi}} n_z}{\tau_{\mathrm{zi}}}\right) u_{i,\varphi,\mathrm{interp}})$	FUN1
$\int_0^\rho V' \left(U_{i,\varphi,\mathrm{exp}} + \langle R\rangle \sum_z \frac{m_{\mathrm{zi}} n_z u_{z,\varphi}}{\tau_{\mathrm{zi}}} + \frac{\langle R\rangle^- m_i n_{i-} u^-_{i,\varphi,\mathrm{interp}}}{\tau} \left(\frac{V'^-}{V'}\right)\right) \partial\rho - \int_0^\rho V' \left(U_{i,\varphi,\mathrm{imp}} - \langle R\rangle \frac{m_i n_i}{\tau} - \langle R\rangle \sum_z \frac{m_{\mathrm{zi}} n_z}{\tau_{\mathrm{zi}}}\right) u_{i,\varphi,\mathrm{interp}} \partial\rho$	INTFUN1