



# Center for Extended MHD Modeling (CEMM)

S. Jardin PI  
2001-2015

*a SciDAC activity...*  
*Partners with:*  
TOPS  
ITAPS  
SWIM  
CPES

**GA:** V. Izzo, N. Ferraro

**U. Washington:** A. Glasser, C. Kim

**MIT:** L. Sugiyama, J. Ramos

**NYU:** H. Strauss

**PPPL:** J. Breslau, M. Chance, J. Chen, S. Hudson

**TechX:** S. Kruger, T. Jenkins, A. Pletzer

**U. Colorado:** S. Parker

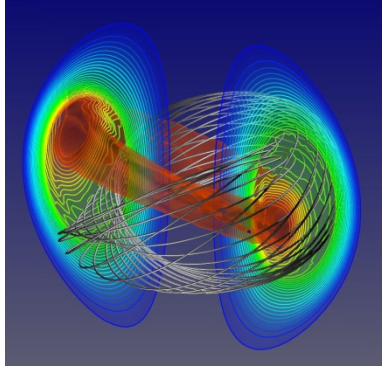
**U. Wisconsin:** C. Sovinec, D. Schnack

**Utah State:** E. Held

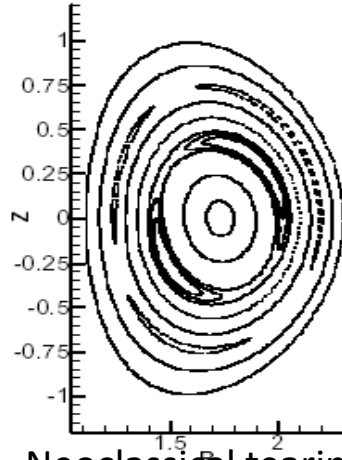
**NIMROD** and **M3D** codes  
(+ new code development  
such as **M3D-C<sup>1</sup>** code)



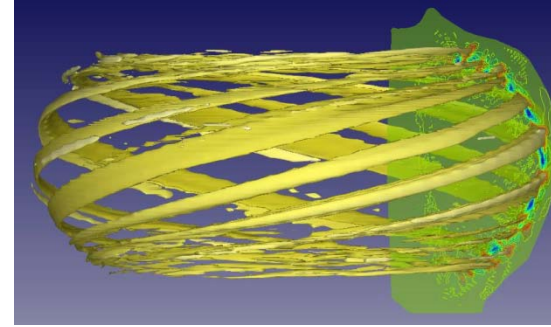
# MHD codes are used for studying a variety of instabilities in tokamaks: Understanding these is very high priority for ITER



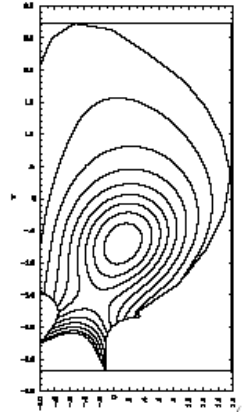
“sawtooth oscillations”



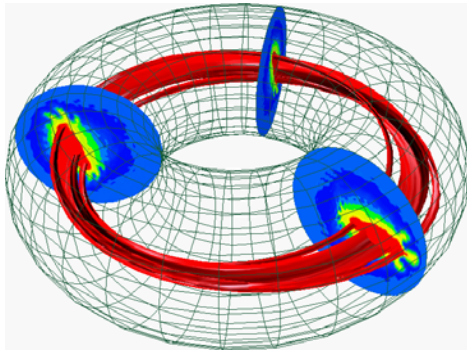
Neoclassical tearing modes and interaction of coupled island chains.



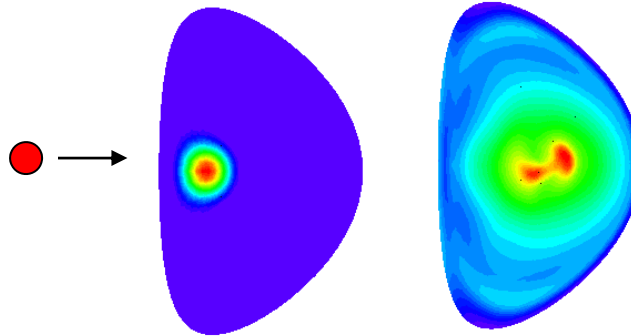
Edge Localized Modes



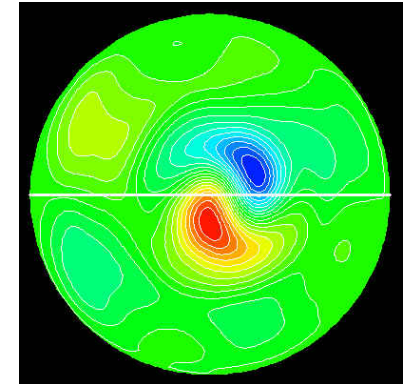
Disruption forces, RE, and heat loads during disruption



Disruptions caused by short wave-length modes interacting with helical structures.



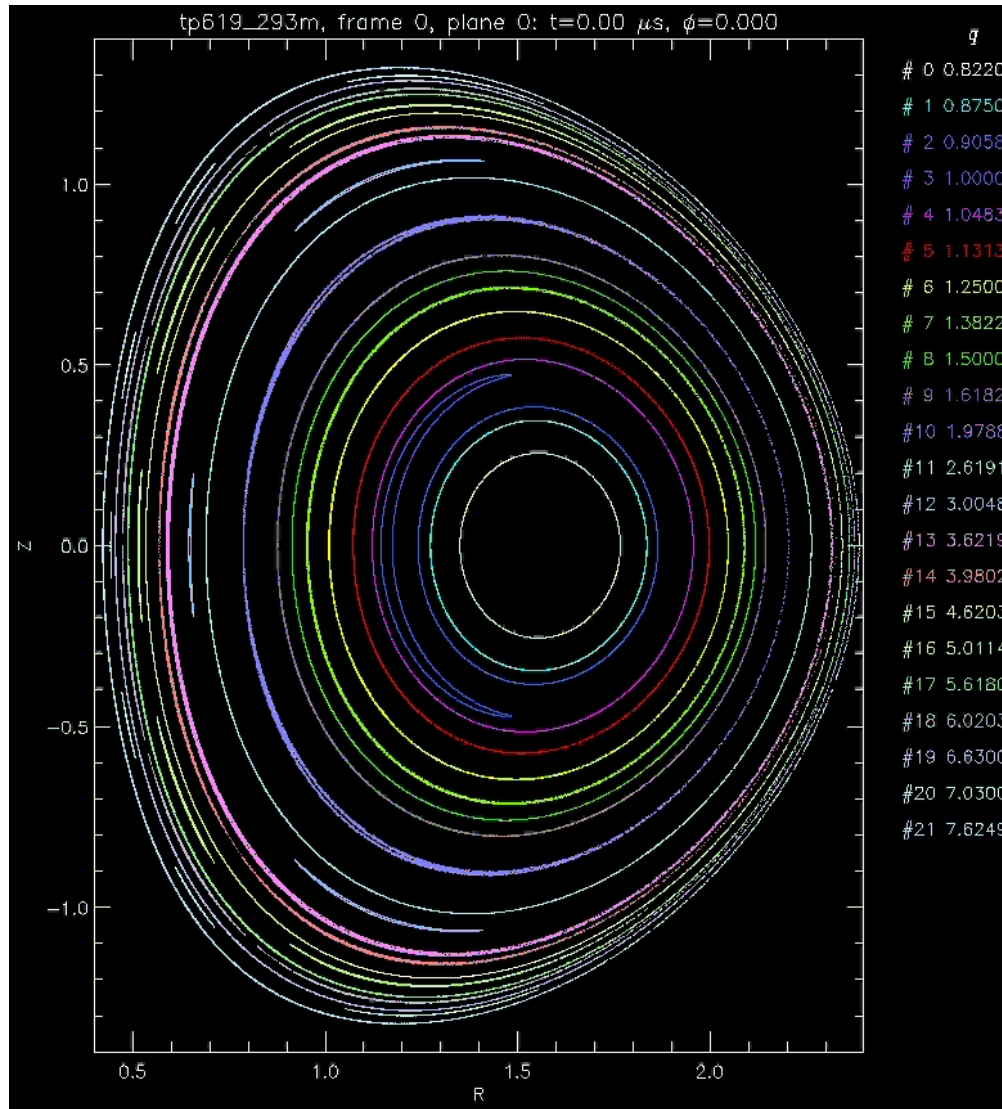
Mass redistribution after pellet injection



Interaction of high-energy particles with global modes

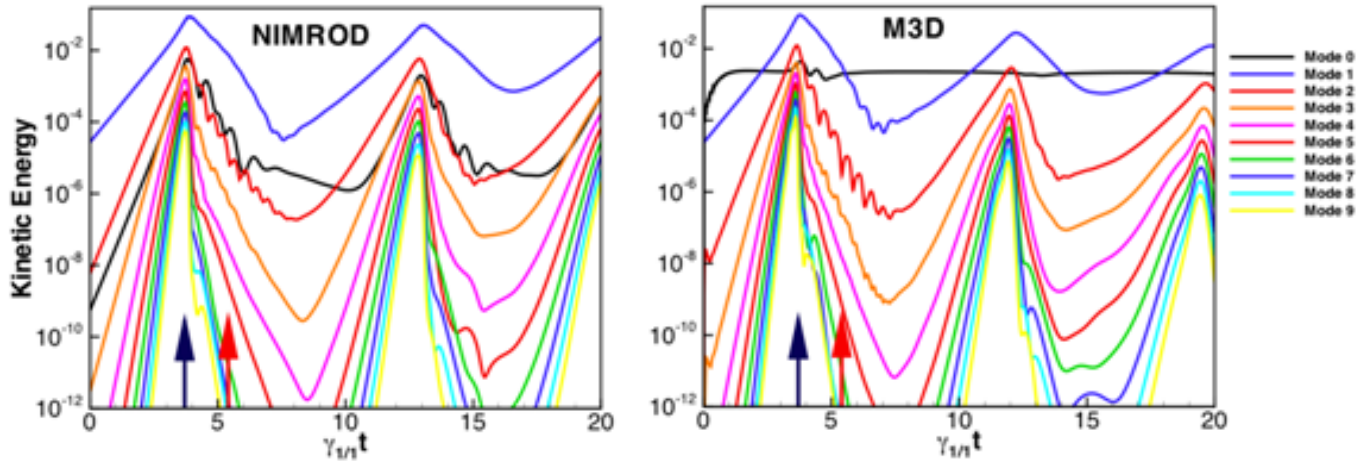


# Several complete sawtooth cycles calculated for a small tokamak (CDX-U) with M3D & NIMROD

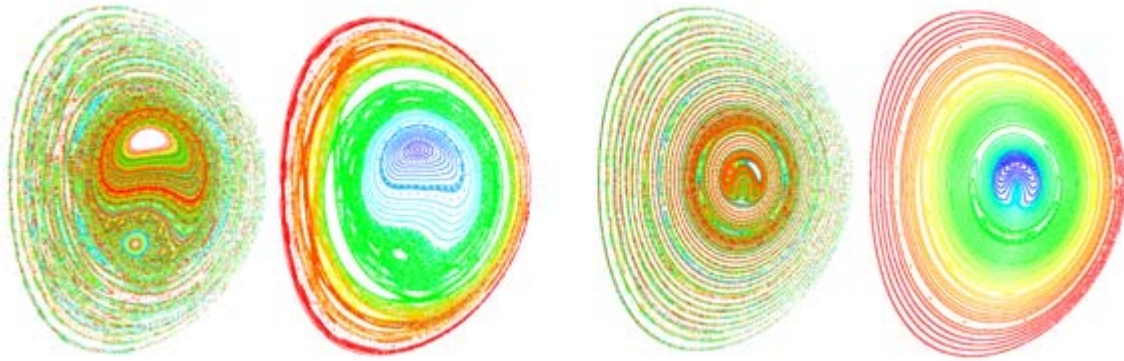


- Poincaré plot shows magnetic field at one toroidal location
- Global mode, but fine-scale structure is important
- Good agreement between M3D, NIMROD, and experimental results
- 500 wallclock hours and over 200,000 CPU-hours
- Goal is to repeat this with parameters of ITER (including energetic alpha particles)

# Excellent Agreement between NIMROD and M3D throughout the nonlinear cycle



Kinetic energy vs time in lowest toroidal harmonics



M3D

NIMROD

M3D

NIMROD

Flux Surfaces during crash at 2 times

# 2-Fluid MHD Equations:

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{V}) = 0 \quad \text{continuity}$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E} \quad \nabla \cdot \mathbf{B} = 0 \quad \mu_0 \mathbf{J} = \nabla \times \mathbf{B} \quad \text{Maxwell}$$

$$nM_i \left( \frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} \right) + \nabla p = \mathbf{J} \times \mathbf{B} - \nabla \cdot \mathbf{\Pi}_{GV} + \mu \nabla^2 \mathbf{V} \quad \text{momentum}$$

$$\mathbf{E} + \mathbf{V} \times \mathbf{B} = \eta \mathbf{J} + \frac{1}{ne} (\mathbf{J} \times \mathbf{B} - \nabla p_e) \quad \text{Ohm's law}$$

$$\frac{3}{2} \frac{\partial p_e}{\partial t} + \nabla \cdot \left( \frac{3}{2} p_e \mathbf{V} \right) = -p_e \nabla \cdot \mathbf{V} + \eta J^2 - \nabla \cdot \mathbf{q}_e + Q_\Delta + S_{Fe} \quad \text{electron energy}$$

$$\frac{3}{2} \frac{\partial p_i}{\partial t} + \nabla \cdot \left( \frac{3}{2} p_i \mathbf{V} \right) = -p_i \nabla \cdot \mathbf{V} + \mu |\nabla V|^2 - \nabla \cdot \mathbf{q}_i - Q_\Delta + S_{Fi} \quad \text{ion energy}$$

Ideal MHD

Resistive MHD

2-fluid MHD

$n$  number density

$\mathbf{B}$  magnetic field

$\mathbf{J}$  current density

$\mathbf{E}$  electric field

$nM_i \equiv \rho$  mass density

$\mathbf{V}$  fluid velocity

$p_e$  electron pressure

$p_i$  ion pressure

$p \equiv p_e + p_i$

$e$  electron charge

$\mu$  viscosity

$\eta$  resistivity

$\mathbf{q}_i, \mathbf{q}_e$  heat fluxes

$Q_\Delta$  equipartition

$\mu_0$  permeability

$S_{Fe,i}$  Fusion power

# General features of tokamak implicit<sup>1</sup> MHD

~ 8-9 variables per element (or mesh point)  $\mathbf{V}$ ,  $\mathbf{B}$ ,  $\rho$ ,  $p_e$ ,  $p_i$

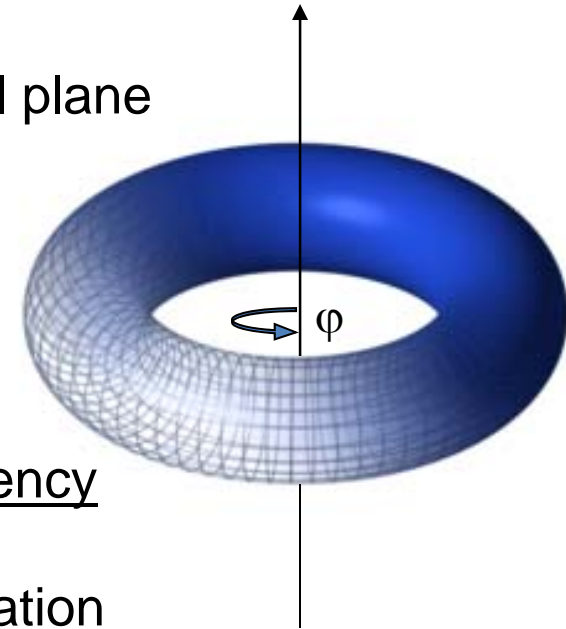
~  $10^4$  -  $10^5$  element DOF per variable per toroidal plane

~  $10^2$  toroidal planes (or Fourier modes)

→  $10^7$  –  $10^8$  DOF per problem

→ Large sparse matrix equations require low latency

→ Typically store all DOF ~ 100 times per calculation



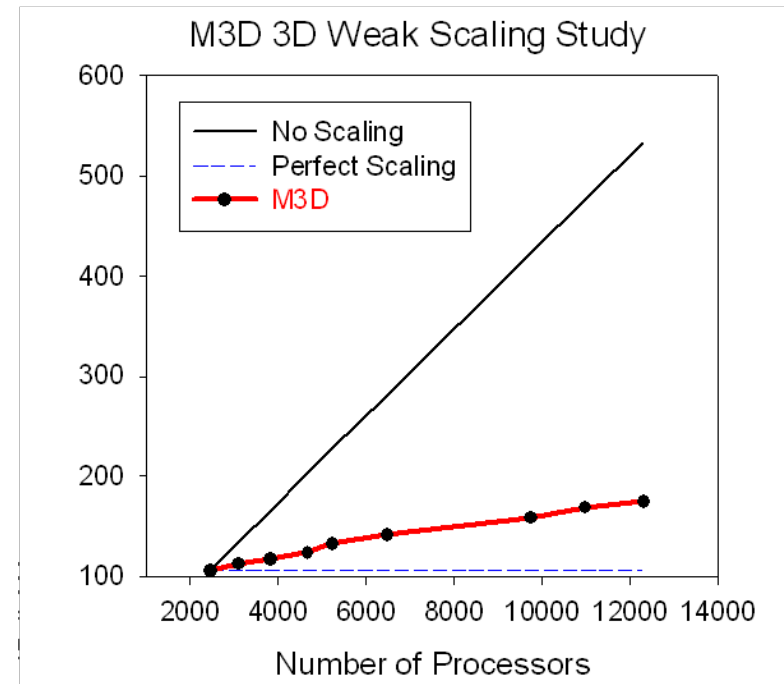
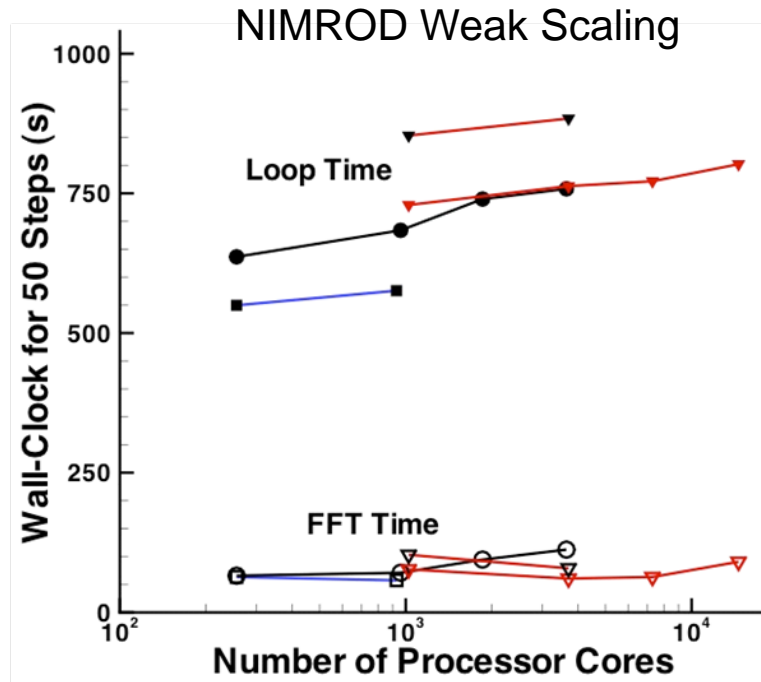
Codes vary in:

- single (big) matrix equation or several smaller equations
- non-linearly implicit (NK), linearly implicit, or partial implicit
- spectral, finite element, or finite differences in toroidal direction

<sup>1</sup>fully explicit time integration not feasible



# Recent scaling studies on Franklin to over 12,000 processors



- Some limited Fourier coupling in preconditioner
- Data and loop reordering

- AMG preconditioned CG
- RCM matrix reordering

M3D scaling properties can be improved by improving data layout...in progress, and being implemented in M3D-C<sup>1</sup>

# Future Trends for 3D MHD Codes

- Need for greater spatial resolution in all three spatial dimensions
  - leads to very large sparse matrix equations
- Wall-clock time / calculation will not decrease as only # processors increases.
  - Will give higher resolution calculations, but will require same or greater wall-clock time.
  - Some calculations now take months.
- Efforts underway to include kinetic effects in MHD codes
  - Hybrid methods already implemented..EP modes, sawtooth
  - Continuum methods being implemented ...neoclassical tearing
- High-order finite elements in 3D may be able to use GPUs for integrations over 3D volume elements.

# CEMM Physics Studies for period 2010 - 2015

- ELMs and ELM control
  - Stabilization by non-ideal effects
  - ELM triggering and ELM-free regimes
  - Effect of magnetic tangle
- Disruption studies and mitigation techniques
  - Wall forces
  - Types of disruptions
  - Runaway electron generation
- Giant sawteeth studies
  - Hybrid kinetic model
- Resistive wall mode
  - Including kinetic effects
- Neoclassical tearing modes
  - Continuum kinetic model
- Error fields and 3D perturbations
  - Include plasma rotation and non-ideal effects
  - RMP effects on ELMs

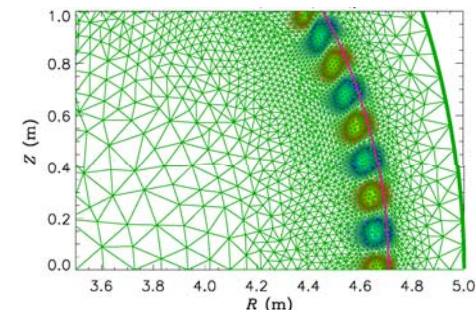
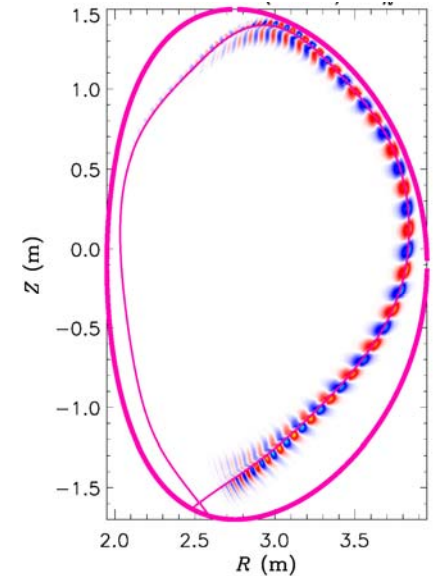
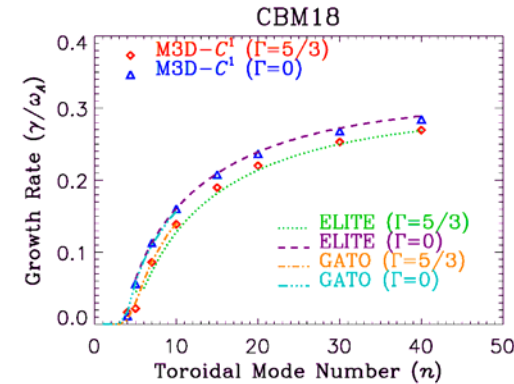
# M3D- $C^1$ has new capabilities

J. Breslau, M. Chance, J. Chen, N. Ferraro<sup>1</sup>, S. Jardin

- Fully implicit, HO FE with  $C^1$  continuity
- Code has now been well benchmarked in linear regime
  - Very accurate
  - Agrees well with ELITE and GATO in ideal limit
  - Can handle separatrix geometry and vacuum region
  - Great flexibility in adaptive meshing
  - S up to  $10^8$  or more, 2F terms, plasma rotation

## NEW Capabilities (2010)

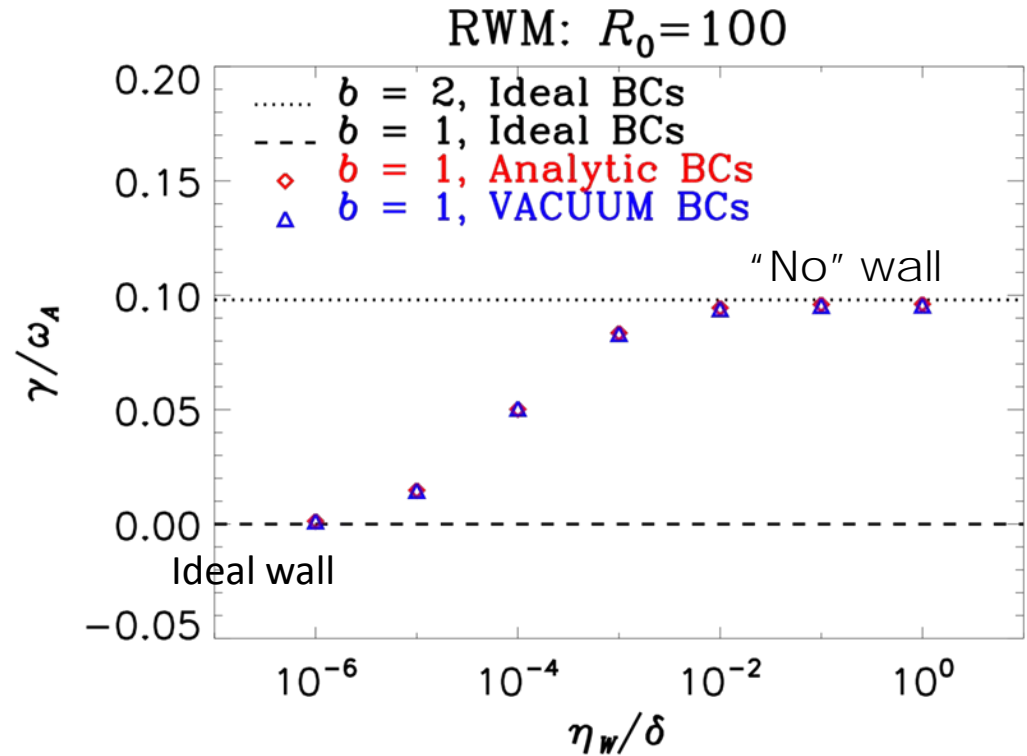
- Resistive Wall Boundary Conditions
- Linear Non-Axisymmetric Response to 3D Fields
- Nonlinear 3D Now Working!



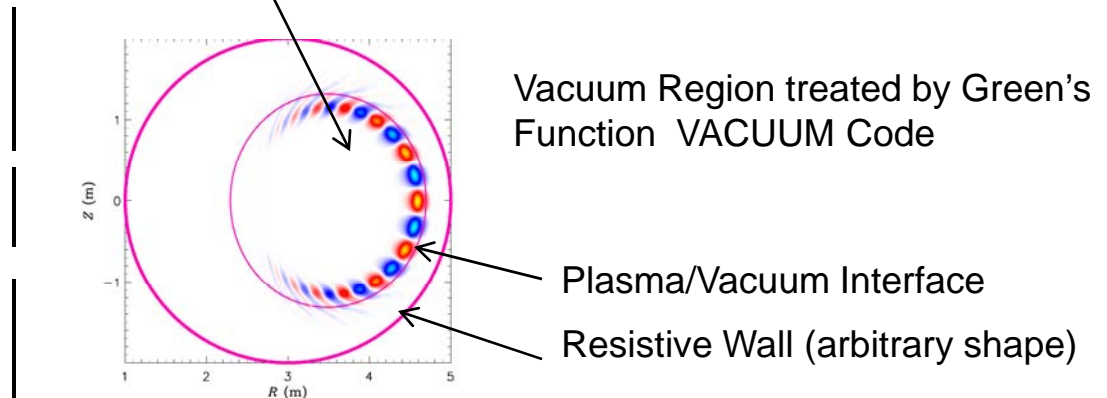


# Resistive Wall Mode Test

- Equilibrium is no-wall ideal unstable when conducting wall is at  $b=2$
- Stable with conducting wall at  $b=1$
- For resistive wall at  $b=1$ , growth rate transitions from ideal-wall limit to no-wall limit as  $\eta_w/\delta$  is increased.
- We also have analytic test cases in large  $R/a$  limit....accurate to 4-5 decimal places



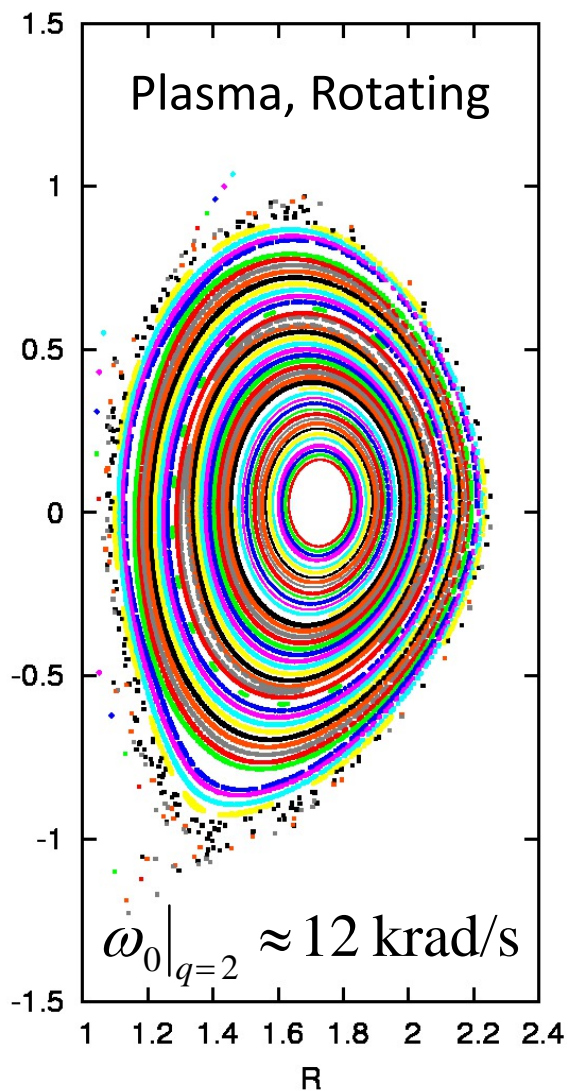
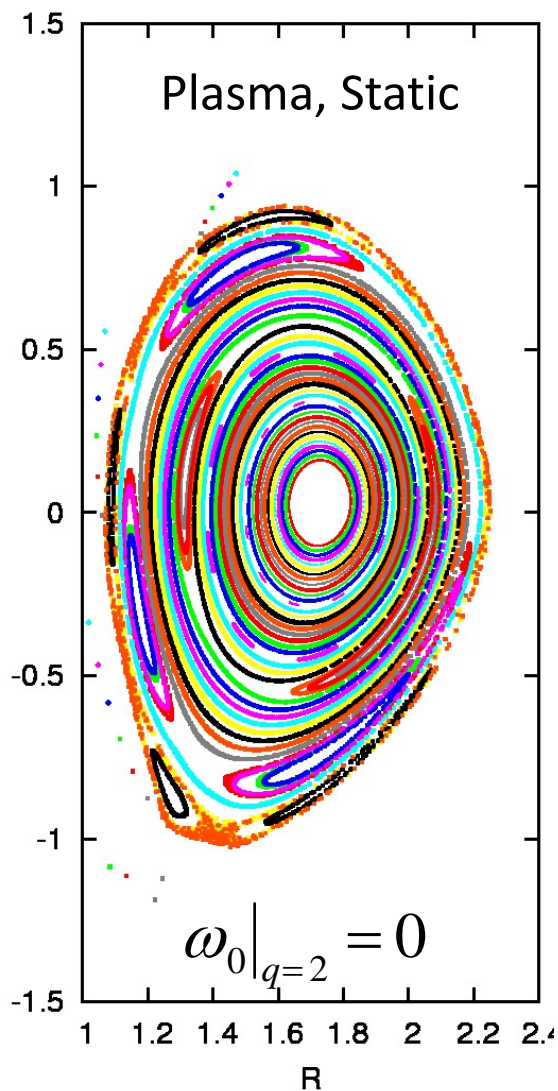
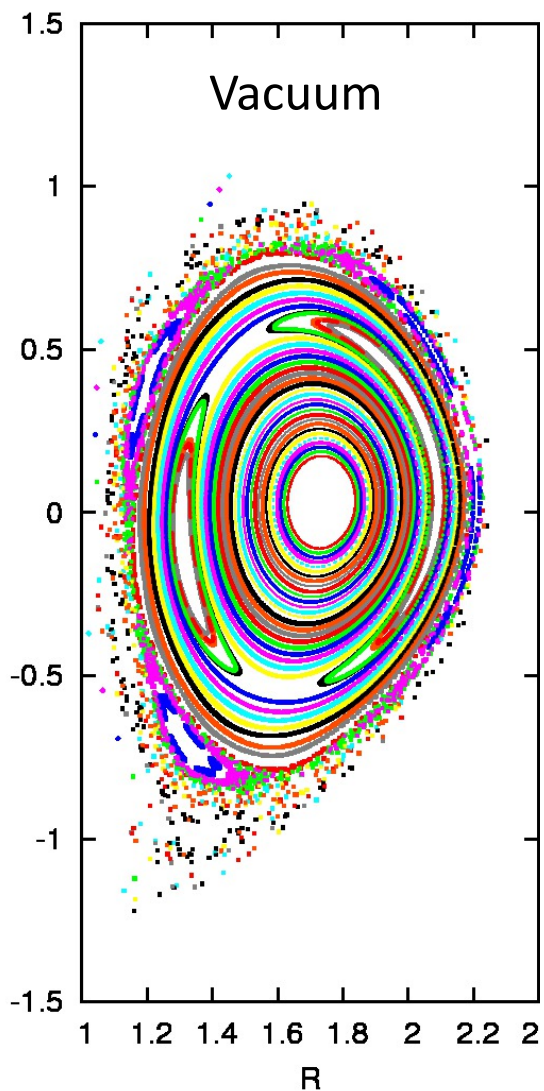
Plasma Region



# Linear Non-axisymmetric Field Response with M3D-C<sup>1</sup>

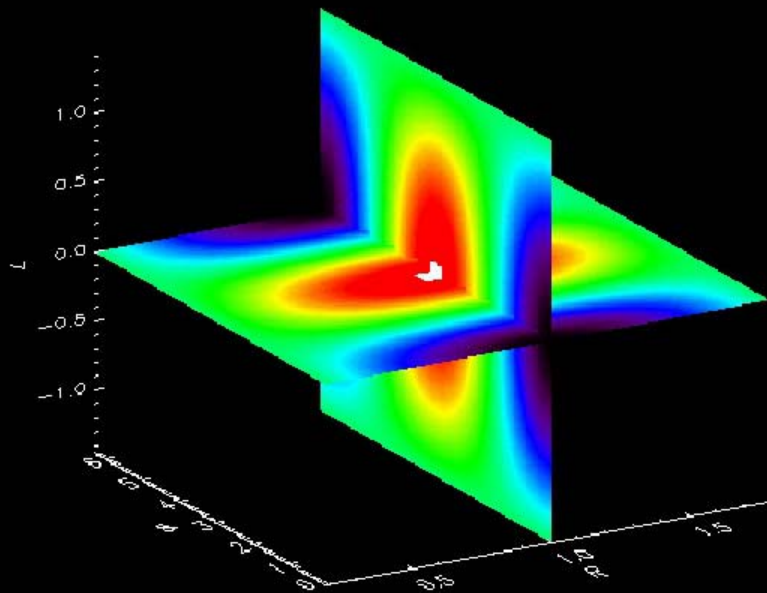
- $B(t) = B_0 + B_1(t)$ 
  - $B_0$  is the axisymmetric equilibrium field
  - $B_1(0)$  is the “vacuum field” from non-axisymmetric coils (I-coils).
- Conducting-wall boundary condition
  - $B$  held constant in time on simulation domain boundary (approximately vacuum vessel)
- Simulation is time-advanced until the steady-state is reached.
- Final  $B_1$  is applied field + plasma response.
- May be a better way to do this; now having active discussions

# Rotation Improves Core Screening but Stochasticizes Edge

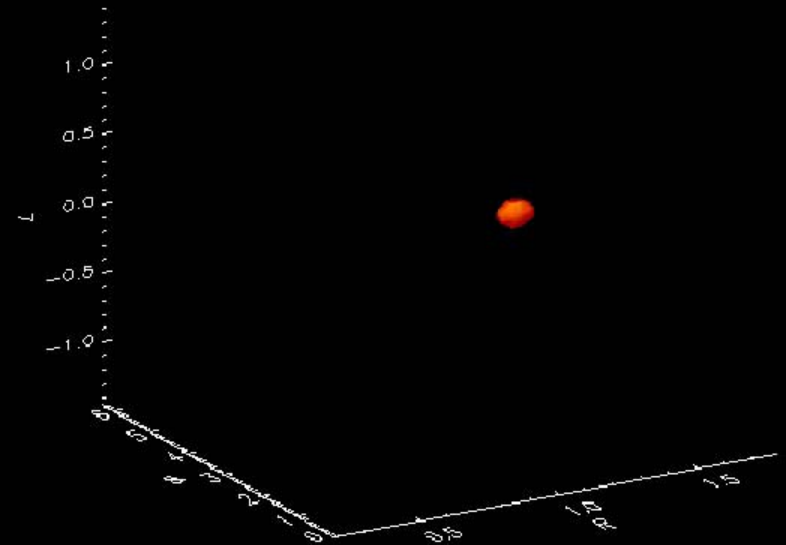


# M3D-C<sup>1</sup> Full 3D Non-linear Capability now Working!

Alfven wave propagating in slab gives correct velocity to 4 decimal digits



Heat pulse propagating in torus gives correct anisotropic behavior





# Summary

- Extended-MHD codes are addressing some of the most important problems for today's tokamaks and for ITER
- Codes solve a coupled system for 8 scalar variables using state-of-the-art implicit techniques
- Codes have  $10^7 - 10^8$  DOF per problem ... leads to large sparse matrix equations. Latency and data movement are key issues.
- Weak scaling has been demonstrated to over 12,000p. This can be improved with better data layout
- Some calculations include kinetic closures which scale even better
- Real-world problems require more resolution than is now practical. This implies more processors, but same or greater wall-clock time
- High-order finite elements may benefit from GPUs for local integrations over volumes.
- M3D-C<sup>1</sup> code shows promise in accuracy and efficiency..nonlinear just starting